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An Economic Analysis of the Production Structure of the Sawmilling Industry in Alberta

by

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A THESIS

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Dedicated

To my father and mother.

ABSTRACT

The purpose of this thesis is to report on an economic analysis of the production structure of the sawmilling industry in Alberta. The rationale for conducting this study arose from broader socioeconomic problems and policy issues of wood supply, employment stability and competitiveness facing the Canadian forestry sector.

Economic analysis of a production structure is generally meant to include the estimation and analysis of economies of scale, factor substitution among inputs, technical change biases in input usages and productive efficiency. The production structure is fairly complex and is approximated by a translog cost function consisting of the prices of labor, capital services, wood and materials inputs and output in the temporal model. The cross-sectional translog model, utilized to study scale economies in the industry, uses the same prices except that energy prices replace material prices.

The production structure is found to be best represented by a nonhomothetic translog cost function in both cases. The selection of the nonhomothetic structure permits significant factor substitutions among pairs of inputs to take place in the sawmills, economies of scale to vary across sawmills and sawmills to be constrained by relative factor prices in expanding their operations. All three phenomena prevail. Also, technical change has been labor and material saving, capital using and wood nuetral

ABSTRACT

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Economic analysis of a production structure is generally meant to include the estimation and analysis of economies of scale, factor specialization among inputs, technical change biased in input usage and productive efficiency. The production structure is fairly complex and is approximated by a flexible cost function consisting of the prices of labor, capital services, wood and materials inputs, and output in the sawmill sector. The first mentioned translog model, the translog ... reveals mostly scale economies in the industry. Also the late period except when capital prices replace materials prices.

The production structure is found to be best represented by a nonhomothetic translog cost function in both cases. The estimation of the nonhomothetic production permits significant factor substitution among groups of inputs to take place in the sawmilling, economies of scale is very strong especially and quality is in particular by relative factor prices in explaining their magnitudes. All these phenomena persist. Also, technical change has been labor and material saving, capital using and wood neutral.

over time.

The time series data are from 1959 to 1981. Results indicate that the derived demand for wood is negative and is the most inelastic, indicating the 'basic-good' nature of this input in the sawmilling industry. The derived demand for labor was the most elastic among all the inputs.

Labor displays substitution relationships with capital and material. The partial substitution elasticity between labor and capital showed a declining trend over time, perhaps reflecting the difficulty of displacing skilled labor by capital. Labor and wood as well as capital and material were both found to be complements. The strength of the complementarity relationship between capital and material, however, has been declining over the years. The substitution relationship found between capital and wood also showed a declining trend since the mid-seventies. Finally, material and wood displayed a fairly stable substitution relationship over the entire 23-year sample period.

Estimation of a Cobb-Douglas stochastic frontier provided some knowledge about technical inefficency in the sawmilling industry. Deviations of observed costs from the minimum costs, the later costs being associated with the stochastic frontier, was revealed to be about 80 percent due to technical inefficiency with the remaining 20 percent accounted for by factors that are beyond the control of the sawmills. Thus the possibility for sawmills to reduce

production costs within the given production structure appears to exist.

The cross-sectional data analysis used 1978 data from 83 Alberta sawmills. The industry consists of very small to very large sawmills. Estimated scale economies revealed that 55 percent of the 83 sawmills analysed had some amount of unexploited scale economies. The possibility of reducing production costs in the industry therefore appears encouraging, first by reducing inefficiency and second by utilizing unexploited scale economies.

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1. INTRODUCTION

The purpose of this study was to conduct an economic analysis of the production structure of the sawmilling industry of Alberta. To carry out this task, a translog cost function was utilized. Several models nested within the most general translog model were generated and tested against the most general model. The idea behind this process was to conduct a series of statistical hypothesis tests in order to select the most appropriate model representing the production structure of the sawmilling industry. Once a model was selected, comparative static analyses were conducted to study various characteristics of the production structure. A sample of cross-sectional data was also utilized to study scale economies in the sawmilling industry.

1.1 Discussion of the Problem

Forests have played a significant role in the development of Canada. Today the forests products industry (lumber,plywood, paper and allied products) ranks fourth in terms of its contribution to gross domestic product. Furthermore, the forest industry attracts significant foreign exchange.

Much of Canada's vast land mass is covered by forests, some of which are commercially available and some of which are not. For a long time this renewable natural resource was thought to escape depletion in supply and may have

contributed to current excesses from harvest rates which are high in relation to replenishment rates. Exploitation of timber in recent times has had to be conducted in less accessible areas. Production costs have in turn increased.[1] The impact of increased production costs can be very significant. Increased production costs make Canada's position in international markets for forest products less competitive relative to her trading partners. In the domestic economy the impact is felt further in terms of reduced income and employment effects.

The debate over whether or not there is an abundant stock of forests in Canada has started to receive serious attention from both forest economists and policy makers. Past research efforts in the forest sector appear to have focused primarily on the biological aspects of forestry with little emphasis given to the socio-economic issues related to the sector.

A recent report (Pearse et al. 1984) points out that the problems faced by this sector are basically economic in nature. The same report also attempts to identify the magnitude of the situation faced by the forest industry sector of Canada in the following manner.

> The economic strain resulting from these trends in Canada and the subsequent loss of comparative advantage and competitiveness on world markets will be further aggravated by relatively stable (e. g., Europe) or slightly rising (e.g., United States) real costs for some of our major trading partners. (Pearse et al., 1984,p. 4).

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[1]See Pearse et al. (1984).

Means to make this sector competitive in international markets on the one hand, and make this sector a viable source of income and employment on the other, stand out as problems that deserve serious attention. Many issues are involved which require the classification of the problem situtation into different components. For example, there are issues that go beyond domestic marketing to international marketing of forest products. The costs of future timber supplies and their likely effects on income and employment in the domestic economy are also areas to be addressed by research economists. Economic analysis of the production structure of the industry provides important feedback to policy makers. There are also issues that relate to investment requirements which may be necessary if the competitiveness of this industry can be enhanced by introducing new technology. Given the different issues involved, the classification of the problem into different dimensions becomes justifiable. A study, such as the present one, can deal only with one such issue due to constraints on the researcher's experience, time, finances, and other resources. This study concentrates on one micro level issue; namely, economic analysis of the production structure of the sawmilling industry in Alberta.

Alberta is endowed with an abundance of timber resources which occupies over half of the province's surface area. Compared to other provinces in Canada, the stock of forest resources in Alberta is relatively accessible and

economically exploitable. Reed and Associates (1978)[2] point out that in the south-west part of the province (generally referred to as zone 2) the estimated softwood reserve is about 2.8 million cubic meters out of which only about one-ninth was committed to companies operating in the area in 1978. The report further points out that the remaining volume is believed to be accessible and economically harvestable. Hardwood stands which are accessible are virtually uncommitted in the province. The west central part of the province also contains huge reserves of accessible softwood and hardwood. Alberta appears to have the best prospects among Canadian provinces for expanding the forest products industry and at the same time attracting foreign investments.[3]

The sawmilling industry in Alberta consists of mills ranging from large scale automated and computerized mills to small scale part time operations. For example, in a survey conducted by the Northern Forest Research Centre, Canadian Forestry Services in 1978, output of Alberta sawmills ranged from five thousand foot board measure (fbm) to over 80 million fbm per mill.[4] Compared to other provinces in Canada the sawmills are fairly new in Alberta in terms of large

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[2]See Reed and Associates (1978) for more details on the relative stocks of timber resources across Canada by provinces.

[3]This later issue was pointed out by S. Nilsson in a seminar that was sponsored by the Canadian Forestry Service, Ottawa, April 4, 1984.

[4]See Ondro and Williamson. This information report prepared by Ondro and Williamson provides an excellent cross-sectional view of the forest industry in Alberta.

scale operations and modern technology (Bigsby, 1983).

Over time there has been a trend towards larger sawmills in Alberta as well as in Canada as whole. Ondro and Williamson (1982) reported that in 1972, 42 percent of the total output of Alberta sawmills was produced by 4.2 percent of the sawmills and by 1978-79, 60 percent of the industry's output was produced by 2.3 percent of the mills. In spite of the large number of small sawmills the role played by these smaller sawmills should not be underestimated. Ondro and Williamson (1982) have documented three major roles the smaller sawmills play in the provincial economy. To quote the authors,

> Despite their relatively minor contribution to total output, the small sawmills play important roles in the local economies for three reasons. First, the revenues generated by these operations are an important source of primary and supplementary income to both full-time and part-time independent operators. Second, they are an important source of supply of low-cost lumber in the local economies. Third, they utilize isolated tracts of timber that may be considered uneconomical by larger operators, (p. 11).

In 1978-79, sawmills accounted for 44 percent of the total employment in the provincial forest industry. Also in the same year over 43 percent of the total value of sales of the forest industry originated from this industry. The domestic market for the sawmilling industry's output is fairly small compared to its market in the United States. For example, in 1978-79 only 26.2 percent of the output from the sawmills was sold within the province, compared to 51.3 percent sold in the United States. The remainder was sold in

other provinces across Canada.

The brief discussion above on the sawmilling industry highlights the economic importance of this industry in the provincial economy. The demand for this industry's output is a derived demand - the demand being derived from the furniture industry, the construction industry, other wood using industries in the domestic as well as foreign economies. Except for the two pulpmills in Alberta, virtually all timber harvested in the province is processed through sawmills. Hence, if the forest products industry in the province is to expand its scale of operations and attract foreign capital investments, the role of the sawmilling industry cannot be underestimated.[5]

No study has been conducted that takes an indepth analysis of the sawmilling industry in Alberta. Previous work by Williamson (1983) who conducted a cross-sectional analysis of a sample of the sawmills represents an important start, however. The scope of that work was limited because the production structure analysis was conducted on the basis of only two inputs, labor and capital, specified in the Constant Elasticity of Substitution production function. Bigsby (1983) utilized time-series data and conducted an economic analysis of the sawmills using the translog cost function. The present study goes beyond these earlier works in the analysis of the sawmills' production structure in

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[5]This is not to underestimate the economic role of the two pulpmills in the province. However, the data for the pulpmills are not available at provincial level in Canada because of the small number of such mills in the country.

that it is more exhaustive. Cross-sectional data are also utilized to estimate the returns to scale in the sawmilling industry using the translog cost function approach.

The Alberta sawmills vary considerably in size and type of mill equipment. Sawmills producing in excess of five million fbm annually generally rely on rapid linear feed sawmill technology. Sawmills in the one to five million fbm annual output size grouping use primarily circular saw headrings for log breakdown. Some large mills also utilize debarkers and chippers and thus produce chips as a by-product. There is a decrease in the sophistication of technology as one examines smaller and smaller mills. The incidence of portable mills increases with smaller mill sizes.

1.2 Objectives

An economic analysis of a production structure generally includes estimation and analysis of the following components:[6]

1. returns to scale;
2. input separability;
3. factor substitution;
4. technical change; and
5. productive efficiency.

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[6]See for example, Moroney (1971). Other issues may include the total factor productivity issue and distribution of factor shares. The first issue is seldom taken up at the industry level. Neither issue is taken up in this study.

1. The existence of constant, increasing or decreasing returns to scale characterizing the production process has been an intergral part of most studies that analyse production structures, especially in the context of individual firms or industries. The estimate of the scale economies can provide valuable insights to policy makers as to the future growth potential of the unit of analysis. For example, if scale effects in an industry are large and the domestic market for the industry's output is small, exploration of foreign markets could become a worthwhile venture. Scale economies have long run implications for the structure of production, i.e., reorganisation.

2. Separability is a structural property in a production model. It allows the specification of a production process in terms of subsets of the total set of possible inputs. It further facilitates empirical estimation as well. Over time the relative importance of certain inputs change in the production process in response to changes in relative factor prices. Separability tests provide an idea on whether or not inputs can be aggregated. Separabilty between inputs, if it exists, allows decision making to proceed in two stages. In the first stage aggregate expenditure on different inputs can be determined. In the second stage the relationship between inputs in each

subgroup can be analysed in submodels.

> Separability is of direct economic interest, implying uniform or invariant behaviour of certain economic quantities, and allowing decentralization in decision-making. It is also of critical interest in the specification of functional forms ...... (McFadden 1978).

Recent developments in flexible functional forms allow the statistical testing of separability issues for any given production process. Furthermore the separability issue has implications about elasticities of substitution.

3. Substitutability is another major issue and concerns the degree to which one factor input can be substituted for another. Since substitution is defined in terms of factor proportions (or factor prices), cost functions can be utilized to study the behaviour of factor proportions (or factor shares).

4. Technical change can also be characterized in a cost function. Usually, certain hypotheses are made about technical change in a production process, which are then tested empirically.

5. Recent developments in the estimation of stochastic frontier functions enable the analysis of technical and allocative inefficiencies for firms in an industry or over time. The econometric issues involved in estimating stochastic frontiers are fairly complicated but possible, at least in the case of technical inefficiency. Knowledge about productive efficiency (technical and allocative efficiencies) can provide insights into possible enhancement of efficiency.

The above issues form the objectives of this thesis in the context of the sawmilling industry in Alberta. An attempt is made to provide results that could be treated as valuable inputs for policy decisions.

1.3 Method of Analysis

The method of analysis adopted in this thesis is grounded in the neoclassical theory of production. In particular, the assumption is made that sawmill output is produced by combining inputs of labor, capital, materials and wood. The analytical approach to represent the production structure of an industry of firms goes beyond the confines of the Cobb-Douglas and the Constant Elasticity of Substitution production functions to a more flexible form of production function, namely the transcendental logarithmic, or translog, function.

Developments in duality theory allow the specification of a production technology either by a production function or a cost function. Certain interesting properties associated with cost functions make the analysis of production structures more appealing from a cost function point of view. Furthermore by selecting flexible form functions many hypotheses that relate to different aspects of the production process can be tested which were not possible with the older functional forms. The required econometric techniques to estimate flexible functional forms are also utilized.

1.4 Outline of the Thesis

Chapter 2 provides the analytical foundations for this thesis. The translog cost function is discussed at length in this chapter and is compared with the Cobb-Douglas and Constant Elasticity of Substitution production (CES) functions. Chapter 3 contains a discussion the econometric issues involved in estimating the translog functions and some test statistics that are utilized to test certain hypotheses related to the production structure. Chapter 4 is devoted to a discussion of the time series data set employed in this study. Discussion of the results is the topic of chapter 5. Chapter 6 is an extension of chapter 5, wherein the stochastic cost frontier concept is discussed. Attempts made to estimate and analyze some interesting issues concerning technical and allocative efficiencies are contained in this chapter. In chapter 7 cross-sectional data obtained from the Northern Forest Research Centre, Canadian Forestry Services are utilized to estimate and analyse economies of scale in the sawmilling industry. The thesis concludes with the summary and conclusions chapter.

2. ANALYTICAL FRAMEWORK

2.1 Introduction

For several decades, the Cobb-Douglas function was virtually the only function that economists used to characterize the production structure of firms, farms or industries. In the early sixties the Constant Elasticity of Substitution (CES) function was introduced into the economics literature. This latter function overcomes some of the limitations inherent in the Cobb-Douglas function, but has problems of its own. In the past estimation using nonlinear (CES) functions proved to be more difficult than the Cobb-Douglas function, especially in the absence of high speed computers.

Recent developments in duality theory allow the specification of a production technology by either a production or a cost function so long as the cost function satisfies the regularity conditions postulated by neoclassical theory of production. Several different topics are also contained in this section. The transcendental logarithmic function is introduced and it is discussed in some detail. The flexible characteristics of this function are discussed in relation to the production process.

2.2 Inflexible Form Production Functions

Among the inflexible[7] form production functions that have had extensive applications to study the production structure of firms or industries are the Cobb-Douglas, the Constant Elasticity of Substitution production function (CES) and the fixed factor proportion, or Leontief production function. These three forms are the most common.

The Cobb-Douglas production function was derived from an empirical relationship observed whereby the total wage bill was proportional to output. Initially the transformation (input) parameters were always specified to sum to one, thus characterizing constant returns to scale. Later other parameters were specified for the inputs of labor and capital, thus making it possible to test statistically the validity of the constant returns to scale hypothesis. Further modifications were made by generalizing the production function to more than two inputs. Given that the production function can be easily estimated, especially after logarithmic transformations of both inputs and output are made, its use in economics remained unchallenged until the 1960's.

However, economists were not totally satisfied with certain features implicit in the Cobb-Douglas production function. For example, the elasticity of substitution between inputs is always restricted to unity. More importantly, by using the Cobb-Douglas production function,

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[7]The meaning of inflexible will be made clear below.

the feasible set of possible alternative production technologies that may be more close to the true underlying production technology cannot be tested.[8]

In 1963, Arrow *et al.* (1963) formulated the CES production function based on their finding that the capital labor ratio among countries varied much more in some sectors than in others and that a linear (log) relationship between value added per unit of labor and the real wage existed. In other words, output per unit of labor was seen to be a changing proportion of the real wage rate. Utilizing cross-sectional data for 24 industries from various countries, they found that the elasticity of substitution between labor and capital was significantly different from unity which led Arrow *et al.* to reject the Cobb-Douglas specification as an adequate description of a production function. The authors then specified the elasticity of substitution to be a constant and were also able to demonstrate that the Cobb-Douglas and Leontief production functions were special cases of the CES production function.

Nevertheless, the CES production function, in spite of the less restrictive a priori assumption of constant elasticity of substitution, has certain limitations. The function is intrinsically nonlinear and hence estimation is not as easy as in the Cobb-Douglas case. Estimation is generally carried out in three stages, even though now it can be estimated directly with the help of computer

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[8]See Walters (1963) for an excellent survey on the Cobb-Douglas function.

programs.[9] Beyond the case of two inputs, the CES function gets very complex and estimation becomes difficult. Also, even though the elasticity of substitution is not restricted to unity, it nevertheless is still restricted to being constant.

In reality the true underlying production structure is rarely known and as a result different functional forms have to be selected to study production structures. Given that the true underlying structure is rarely known, selection of functional forms that are less restrictive appears to be more desirable. In other words functional forms that do not a priori restrict the parameter estimates and other comparative static summary measures and that allow different hypotheses to be tested, are more desirable. The Cobb-Douglas and the CES (as well the Leontief) functions all fail to be flexible in the sense discussed above and hence may be labelled as inflexible form production functions.[10]

The need for flexible form functions should therefore be apparent from the limitations inherent in the two classes of production functions discussed above. During the last decade or so new functional forms have been introduced in the literature. All of these new functions, (translog,

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[9]See Nerlove (1967) for a survey of CES production function and its applications.
[10]Flexible as well as inflexible functions are, however, assumed to fulfill all properties of a well behaved neoclassical production (or cost) function. More of this aspect of functions is discussed below in the context of cost functions.

generalized Leontief, and generalized Cobb-Douglas to name a few) have one thing in common; namely, they are far less restrictive in a manner described above than the older functions. Among the major flexibilities associated with the flexible form functions are as follows: (1) the elasticities of substitution are not restricted a priori; (2) complementarity elasticties can be estimated; and (3) sequential hypotheses testing of different models nested within the most general flexible function can be statistically conducted to identify various characteristics of the production structure.

2.3 Cost Function

Duality allows the specification of a production structure either by a production or a cost function so long as the cost function satisfies certain regularity conditions.[11]

A very general cost function (C) can be written as a function of input prices (P_i) and output (Y_j):

$$C = C\ (P_i, Y_j).$$

When j = 1, the cost function is said to be a single output cost function and for values of j greater than one, the cost function represents a joint output or multiple output cost

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[11]The principles of duality will not be discussed here and it is taken for granted that duality exists between production and cost functions. See Shephard (1970), Diewert (1971), or Varian (1978) for details on duality.

function. Since the present study is concerned with a single output, namely lumber, the single output cost function can be written as

$$C = C\ (P_i,\ Y). \tag{1}$$

The P_i's refer to input prices of labor (L), capital (K), materials (M) and wood (W).[12] The cost function (eq. 1) is a very general cost function and is said to be well behaved in the neoclassical sense if the following conditions are satisfied.

1. The cost function must be a strictly positive function for positive input prices and positive output levels and implies

$$C\ (P_i,Y) > 0, \text{ for all } P_i > 0,\ Y > 0\ . \tag{2}$$

2. The cost function is a non-decreasing function in input prices. If P_i and P_j are sets of two input prices with $P_i \geq P_j$ then

$$C(P_i,Y) \geq C(P_j,Y). \tag{3}$$

3. The cost function is positively linearly homogeneous in input prices. When all prices double, cost also has to double,

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[12]Chapter 4 contains a detailed discussion on the inputs, prices, output, total cost and other related issues.

$$C(\lambda P_i, Y) = \lambda C(P_i, Y). \tag{4}$$

where λ is a positive scalar.

4. The cost function is concave in input prices and for any scalar λ, $1 \geq \lambda \geq 0$;

$$C\ \{\lambda P_i + (1-\lambda)P_i, Y\} \geq \lambda C(P_i, Y) + (1-\lambda)C(P_i, Y). \tag{5}$$

5. The cost function is a continuous function of prices such that the first and second partial derivatives exist and

$$\partial C/\partial P_i > 0 \text{ and } \partial^2 C/\partial P_i\ \partial P_j < 0. \tag{6}$$

When all of the above conditions are satisfied, the cost function is said to be a well behaved neoclassical cost function. Applying Shephard's lemma to the cost function results in factor demand functions of the form

$$\partial C/\partial P_i = X_i\ (P_i, Y). \tag{7}$$

where in equation (7), $X_i(P_i, Y)$ refers to the ith input demand function.

Given the properties (1 to 5) that all well behaved neoclassical cost functions have to satisfy, certain restrictions on the production technology are implied. Specifically, linear homogeneity in input prices imposes certain restrictions on the production technology (i.e.,

imposes restrictions on the cost function prior to actual empirical estimation to insure reasonable economic behavior). The restrictions imposed a priori on the cost function are as follows:

1. The sum of input costs must add up to total cost which is known as the adding up condition and can be written as

$$\Sigma_i P_i \; X_i(P_i, Y) = C \; (P_i, Y); \qquad (8)$$

2. Cournot's aggregation condition states that a change in the ith input price can lead to a reallocation of the total cost without violating the adding up condition and can be written as

$$\Sigma_j P_j \; \partial X_i(P_i, Y) \; / \; \partial P_i + X_i = 0; \qquad (9)$$

3. Engel's condition states that a reallocation of cost will still fulfill the adding up condition and can be written as,

$$\Sigma_i P_i \; \partial X_i(P_i, Y) \; / \; \partial C_i = 0; \text{ and} \qquad (10)$$

4. Symmetry implies the equality between the cross second order partial derivatives

$$\partial^2 C(P_i, Y) / \; (\partial P_i \; \partial P_j) = \partial^2 C(P_i, Y) \; / \; (\partial P_j \; \partial P_i). \qquad (11)$$

In empirical work a specific functional form has to be selected to represent the cost function (eq.1). Economic theory provides very little guidance to this selection of functional forms except for the restrictions that are implied by the neoclassical theory of production. Furthermore, in view of the earlier discussions on inflexible form functions, it is desirable to select functional forms that can represent fairly complex technologies and yet be fairly simple in specification as well as estimation.

The transcendental logarithmic cost function or the translog cost function for short, developed by Christensen, Jorgenson and Lau (1973) is selected for empirical work. The following section provides a detailed discussion of the various aspects of the translog cost function that are pertinent to the analysis of the structure of production for Alberta's sawmills.

2.4 The Translog Cost Function

The purpose of this section is to introduce the nonhomothetic translog cost function and highlight some of its more flexible features. The objective here is to determine the manner in which structural characteristics of a production process can be studied with the help of a cost function.

The nonhomothetic translog single output, multi-input cost function developed by Christensen, Jorgenson and Lau

(1973) can be written as,

$$\ln C_t = \gamma_0 + \gamma_y \ln Y_t + \gamma_t t + \Sigma_i \gamma_i \ln P_{it} +$$
$$(1/2)\ \Sigma_i \Sigma_j \gamma_{ij} \ln P_{it} \ln P_{jt} + \Sigma_i \gamma_{iy} \ln P_{it} \ln Y_t +$$
$$\gamma_{yt} \ln Y_t\ t + \Sigma_i \gamma_{it} \ln P_{it} t + (1/2)\ \gamma_{tt} t^2 +$$
$$(1/2)\ \gamma_{yy} (\ln Y)^2_t\ . \tag{12}$$

for all i,j = L, K, M, W.

The γ's are the parameters of the cost function, t is time which is used as a proxy for technical change and other variables are as defined previously. Utilizing the restrictions required by linear homogeneity in input prices condition, the following restrictions on the parameters of the translog cost function are implied:

$$\begin{aligned} &(i)\ \Sigma_i \gamma_i = 1; \\ &(ii)\ \Sigma_i \gamma_{ij} = \Sigma_j \gamma_{ji} = 0; \\ &(iii)\ \Sigma_i \Sigma_j \gamma_{ij} = 0; \\ &(iv)\ \Sigma_i \gamma_{iy} = 0; \\ &(v)\ \Sigma_i \gamma_{it} = 0;\ \text{and} \\ &(vi)\ \gamma_{ij} = \gamma_{ji}. \end{aligned} \tag{13}$$

These restrictions (eq. 13) are generally imposed prior to actual estimation of the parameters of the cost function. In other words, the restrictions (eq.13) are maintained hypotheses in most studies that have utilized flexible form functions. However, the Slutsky symmetry restrictions $\gamma_{ij}=$

γ_{ji} in some recent studies have been statistically tested for their validity. If the symmetry restrictions are not valid, the immediate consequence is the non-equivalence between the two pairs of substitution elasticities $\sigma_{ij} = \sigma_{ji}$.[13] According to production theory, $\sigma_{ij}= \sigma_{ji}$. But if $\sigma_{ij} \neq \sigma_{ji}$,then the validity of production theory as well as the data utilized to estimate the substitution elasticities, become questionable.[14]

2.5 Elasticities Of Substitution And Price Elasticities

For the translog cost function, Binswanger (1974B) has derived the Allen-Uzawa partial elasticities of substitution as follows:

$$\sigma_{ii} = \{S_i^2 - S_i + \gamma_{ii}\}/S_i^2 \text{ and} \tag{14}$$

$$\sigma_{ij} = \{ S_i S_j + \gamma_{ij}\}/ S_i S_j, \tag{15}$$

for all i,j and $i \neq j$.

The elasticities are related to the second order coefficients of the translog cost function and the input cost shares (S_i). The elasticities of substitution reflect the sensitivity of change in total cost as a result of a

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[13] See below for the definition of the substitution elasticities in the context of the translog cost function.
[14]Lopez (1980), utilizing a generalized Leontief cost function in the context of Canadian agriculture reports that the null hypotheses of symmetry were rejected at reasonable levels of significance. Also see the paper by Berndt, Darrough and Diewert (1977), where symmetry tests were carried out for several flexible form functions. The symmetry restrictions, if valid, reduce considerably the number of parameters that have to be estimated.

change in the quantity of factor input i when the price of factor j changes and all other prices and output are held constant.[15] Complementarity are indicated by negative values of(σ_{ij}). These partial elasticities of substitution (PES) are not restricted to any particular values but can vary unlike those in the Cobb-Douglas or the CES cost functions, where such elasticities are constant a priori. The Cobb-Douglas cost function is the special case of the translog cost function when all γ_{ij} = 0. When all γ_{ij} are set equal to zero in equation 15 above, the resulting elasticity of substitution can be seen to equal unity.

The own and cross-price elasticities were also derived by Binswanger (1974B) as follows:

$$\eta_{ii} = S_i\sigma_{ii}, \text{ and} \tag{16}$$

$$\eta_{ij} = S_j\sigma_{ij}. \tag{17}$$

Note that even though there is symmetry between the substitution elasticities, symmetry does not exist between cross-price elasticities.

When separability among input uses occur, the partial elasticities of substitution are also affected. The case of global separability implies complete independence among all

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[15]Binswanger (1974B) derived the above formulas without assuming constant returns to scale. Note that the own substitution elasticities have no economic meaning but must satisfy the constrainst (S_i σ_{ij} = 0), see Binswanger (1974B).

inputs utilized and the form of the function in this case is the Cobb-Douglas function. Weak separability on the other hand implies a partial or fixed relationship among input use. In both cases the partial elasticities of substitution are affected.[16]

2.6 Nonhomotheticity and Homotheticity

The translog cost function appearing in equation 12 is called a nonhomothetic cost function. The nonhomothetic translog cost function is the most general of all the translog cost functions. By imposing restrictions on the parameters of the nonhomothetic cost function, several different cost functions can be generated. The imposition of the restrictions is carried out in a sequential order such that starting from the most general cost function, more and more restrictive cost functions can be generated. Each cost function generated in this sequential manner implies different production characteristics as outlined below. Statistical testing is carried out to select, among the various cost functions, the one that most appropriately represents the production structure.[17]

A nonhomothetic cost function implies certain structural characteristics of the production structure.

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[16]Separability issues are discussed in greater details below. The separability results presented below can be utilized in equation 15 to have an idea of how the substitution elasticities will be affected. For more details reference can be made to Berndt and Christensen (1973B)

[17]The statistical aspects are postponed until the next chapter.

First a nonhomothetic cost function implies a nonlinear expansion path. In other words, the marginal rate of technical substitution varies along the expansion path which is possible only if the expansion path is nonlinear. The ratio of any two cost share equations (expansion path) is not independent of the level of output. However, the cost share equations by themselves are linear. For the nonhomothetic translog cost function, the ratio of two cost share equations is given by

$$S_i/S_j = (\gamma_i + \Sigma_i \gamma_{ij} \ln P_i + \gamma_{iy} \ln Y + \gamma_{it} t) / (\gamma_j + \Sigma_j \gamma_{ij} \ln P_j + \gamma_{jy} \ln Y + \gamma_{jt} t). \tag{18}$$

Equation (18) is clearly nonlinear and depends on the level of output as well since, along the expansion path, relative prices are held constant.

The second structural characteristic associated with the nonhomothetic cost function is that scale economies are not independent of factor prices. This means that firms or an industry can be constrained by factor prices in expanding their operations (Denny 1974).

For the translog cost function, the elasticity of total cost with respect to output is given by

$$\partial \ln C/\partial \ln Y = \gamma_y + \gamma_{yy} \ln Y + \Sigma_i \gamma_{iy} \ln P_{it} + \gamma_{yt} t. \tag{19}$$

Scale economies (SE) can be defined as unity minus the cost

elasticity,

$$SE = 1 - \partial \ln C / \partial \ln Y. \tag{20}$$

As defined in equation 20, SE is in percentage terms, with positive values referring to scale economies and negative values referring to scale diseconomies. The SE as defined in equation 20, is not independent of the technological change parameter and, as a result, cost reductions that may have resulted from technological progress over time, cannot be separated from cost reductions due to scale economies. Thus scale economies obtained from the above relationship have to be interpreted with caution. Christensen and Greene (1976) have argued that scale economies are, therefore, better studied using cross-sectional data, since in cross-sectional data the implicit assumption is that technological change is held constant.

Closely related to the nonhomothetic concept is the concept of homotheticity. All homogeneous cost functions are homothetic, but not all homothetic cost functions are homogeneous. Isoquants corresponding to a homothetic cost function are radial blow ups from the origin implying that the distance between isoquants changes as the level of output increases along the cost minimizing expansion path. The linearly homogeneous cost function is characterized by isoquants that are equally spaced, (i.e., constant returns to scale). Secondly, the ratio of two cost share equations

is independent of the level of output in the case of both homothetic and homogeneous cost functions. In terms of the translog cost function, the homothetic function is obtained by setting the coefficients of the input-output price parameters, $\gamma_{iy} = 0$. [18] On the other hand, homogeneity implies in addition to homotheticity, the restriction that $\gamma_{yy} = 0$. By substituting $\gamma_{iy} = 0$ for all i in equation 18 and $\gamma_{yy} = 0$ in equation 19 above, the resulting equations can be readily examined.[19]

In the literature, a distinction is also made between weak and strong homotheticity. Weak homotheticity refers to the case when changes in output affect the demand for factor inputs in a proportional way. In other words, weak independence is said to exist between input demand and the level of output. The case of strong homotheticity occurs when changes in the level of output have no effect on the demand for inputs (i.e., a strong independence is said to exist between input demand and output). These restrictions are nonlinear in nature and are difficult to impose in the nonhomothetic cost function.

The weak homotheticity assumption implies that nonlinear restrictions $\gamma_{iy} = \gamma_i \delta_y$ for all i; and δ_y is an

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[18]For more details see Silberberg (1978), Denny (1974) Denny and May (1978), Christensen and Greene (1976) and Nadiri (1982). Notice that in the nonhomothetic case we have $\Sigma_i \gamma_{iy} = 0$ whereas in the homothetic case $\gamma_{iy} = 0$, for all i.
[19]A homogeneous cost function must not be confused with a cost function that is linearly homogeneous in input prices. All cost functions are linearly homogeneous in input prices, but not necessarily homogeneous.

unknown constant.[20] The strong homotheticity assumption implies the restriction that all $\gamma_{iy} = 0$.[21] Production functions homogeneous of degree one are characterized by isoquants that are equally spaced (i.e., constant returns to scale) in the input space.

The other nested models within the nonhomothetic translog model can be obtained by imposing further restrictions on the parameters of equation 12. A nonhomothetic translog cost function that exhibits unitary elasticity of substitution among inputs requires setting all $\gamma_{ij}= 0$. This same restriction when imposed on the homothetic function yields a homothetic unitary elasticity of substitution production function. The Cobb-Douglas cost function is obtained finally when all second order parameters are set to zero on the homogeneous cost function.

2.7 Separability

Separability is an important concept in the study of production structures. Separability is a structural property in production and allows the specification of a production

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[20]The translog function is an approximation to an unknown twice differentiable technology. The paramaters of the translog function are derivatives in the Taylor's series expansion of the general function evaluated at a particular point, which in the case of the translog cost function are at 0 for all variables. See Berndt and Christensen (1973A, 1973B), Denny and Fuss (1977) and Binswanger (1974B) for details.

[21]In the literature the test for weak and strong homotheticity is seldom conducted. See for example, the papers by Lopez (1980) and Denny and May (1978) among a few. Taher (1983), however, conducted the tests for both weak and strong homotheticity. In this thesis, these tests are carried out and are reported in Chapter 5.

process in terms of subsets of the total set of inputs. In terms of the cost function, separability among inputs allows the specification of a cost function in terms of subcost functions, and each subcost function can be either linear or quadratic.[22] Separability further imposes restrictions on the possible substitutability among factor inputs.

> Separability is of direct economic interest, implying uniform or invariant behavior of certain economic quantities, and allowing decentralization in decision making. It is also of critical interest in the specification of functional forms. (McFadden 1978).

If separability holds, then inputs can be partitioned into subgroups so that some form of independence among the uses of inputs in each subgroups exists. Depending upon the type of independence among input usage, each subgroup can have one or more further subgroups within it and inputs purchased within each subgroup can be written as a function of group expenditure and prices. An important implication resulting from separability arises. Separability among inputs imposes restrictions on the elasticity of substitution and, since detailed expenditure on inputs can be related to group expenditure and prices alone, econometric estimation can be conducted from smaller numbers of variables to analyse aggregate expenditure.

In the literature, three forms of separabilty concepts exist; namely, global, weak, and strong separability. When inputs used in a production process are entirely independent

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[22]See Denny and Fuss (1977) for details on this issue. Also see Berndt and Christensen (1973A, 1973B).

of each other, global separability among inputs is said to exist. Stated differently, global separability testing is equivalent to testing the null hypothesis of a Cobb-Douglas functional specification as appropriate against the alternative hypothesis that it is not appropriate. The relation between input usage could also be weak or strong.[23] Weak independence implies that input use in one subgroup would be related to input use in another subgroup only in a fixed manner. Thus the marginal rate of technical substitution between two inputs in a subgroup is weakly independent of input use in another subgroup. In the case of strong separability, not all inputs will exhibit a strong independent relationship with each other, whereas in the case of global separabilty, all inputs will exhibit a strong independent relationship between each other.

When separability of material inputs exist a very general cost function of the type $C = C(P_i, Y)$, can be written as

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[23] Berndt and Christensen (1973A, 1973B) use the terms linear and nonlinear separability restriction. Denny and Fuss (1977) use the terms weak and strong separability restrictions. Fuss and Denny (1977) show that when the translog function is considered to be an approximate representation of a twice differentiable unknown technology, the separability restictions developed by Berndt and Christensen (1973B) are too severe. They develop the necessary restrictions in the case where the translog function is assumed to be an approximation to an unknown technology. In this thesis the translog function is assumed to be an approximation to a twice differential unknown production technology. See the above articles for more details.

$$C = C\{ C_m(P_m),\ P_i,\ Y\},\ \text{for } i = L, K \text{ and } W, \tag{21}$$

and the cost function is weakly separable in P_m. The subcost function C_m consists of materials inputs, and the other input prices are not arguments of the subcost function. By applying Shephards lemma to this separable cost function, the material input demand function is obtained,

$$X_m = \partial C/\partial P_m = (\partial C/\partial C_m)\ (\partial C_m/\ \partial P_m). \tag{22}$$

The effect of a change in the prices of L, K, W on the demand for material is obtained by partially differentiating X_m with respect to the respective input prices. As an illustration, consider the effect of a change in the price of capital,

$$\begin{aligned} \partial X_m/\partial P_k &= \partial^2 C/(\partial P_m\ \partial P_k) \\ &= \{(\partial C_m\ /\partial C_m\ (\partial^2 C\ /\partial C_m\ \partial P_k)\} + \\ &\quad \{\ (\partial C/\partial P_m)\ (\partial^2 C_m\ /\ \partial P_m\ \partial P_k)\}. \end{aligned} \tag{23}$$

The subcost function $C_m(P_m)$ does not contain the price of capital as an argument and hence the derivative $\partial^2 C_m/\{\partial P_m\ \partial P_k\} = 0$, and as a result the effect of a change in the price of input i (for i= L,K and W) on the demand for material is generally given

by

$$\partial X_m/\partial P_i = \partial^2 C / (\partial P_m . \partial P_i)$$
$$= \{(\partial C_m / \partial P_m)/(\partial^2 C / \partial C_m \partial P_i)\}. \qquad (24)$$

The interpretation of equation 24 is as follows. The effect of a change in the price of labor on cost can be decomposed into two products when weak separability between material and labor exists. There is a first round effect, namely, when the price of labor increases the relative price of material decreases which allows producers to demand more material relative to labor. This effect, when more materials are demanded, results in a change in the subcost function and is captured by $\partial C_m/\partial P_m$. The second round effect is then transmitted to total cost and is captured by the second term, which can be interpreted in the following manner. The change in the price of labor affects not only the subcost function as illustrated above, but also the quantity of labor demanded which further affects total cost. The price of other inputs and output are held constant. Other cases of weak separability can be derived in a similar manner.

The case of strong separability implies total independence between material and other inputs utilized and hence the changes in the price of other inputs have no effect on the demand for material. Thus,

$$\partial X_m/\partial P_k = \partial^2 C/\{\partial P_m \, \partial P_k\} = 0 \text{ for } i = L,K,W. \tag{25}$$

When the cross-product terms of particular input coefficients are all zero the corresponding input is said to be strongly separable. Only a fixed relationship exists between the input that is strongly separable and all other inputs.

In terms of the translog cost function, the global, weak and strong separability conditions illustrated above can be indentified by imposing certain restrictions on the first and second order derivatives of the translog cost function.[24] The derivatives of the general cost function can be identified with the paramaters of the translog cost function. When these derivatives are identified with translog cost function parameters, the resulting weak and strong separability restrictions are respectively,

$$\gamma_{ij} = \gamma_i \, \rho_j, \text{ and } \gamma_{ij} = 0, \tag{26}$$

where in equation (26) ρ_j is an unknown constant.[25]

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[24]The details are contained in Berndt and Christensen, (1973A, 1973B), Denny and Fuss (1977).

[25]In estimation the weak separability restrictions have to be manipulated to get rid of the unknown term ρ. More on this is taken up in chapter 3 while discussing the Wald test statistic.

2.8 Analysis Of Technical Change Bias

Technical change arises from the recognition that the set of all known production techniques may change over time. Technical change in the neoclassical production framework is represented by an inward shift of the isoquants when both inputs and output are held constant. This inward shift of isoquants may arise due to technical change in one or more inputs made possible through research and developement, cumulative experience, better management, or simply, the passage of time.

In the nonhomothetic translog cost function, the time variable, t, is used as a proxy for technical change. There are several definitions of technical change that can be found in the literature.[26] The most widely used definition is the Hicksian concept of technical change. This concept is utilized in this study as well.

Technical changes are Hick's neutral (all $\gamma_{it}=0$) if the marginal rate of technical substitution between two inputs remains constant at increasing levels of output. This neutrality implies that the marginal rate of technical substitution does not change over time, even though output may increase. Likewise technical change is said to be factor i saving if the marginal rate of technical substitution of input i for j shows a decline over time. A major drawback of the Hicksian definition of technical change is that it can be measured for only (n-1) factors because technical change

[26]See Nadiri (1970) for elaborate discussions on technical change and various definitions.

is defined in terms of factor ratios and one input acts as a numeraire. The suggestion has been made by Binswanger (1974A, 1974B) that technical change be defined in terms of factor shares rather than factor proportions. This suggestion can be readily translated in terms of the translog cost function. Shephard's lemma applied to the translog cost function generates the cost share equations S_i

$$S_i = \gamma_0 + \Sigma_{ij}\gamma_i \ln P_i + \gamma_{iy}\ln Y + \gamma_{it}t. \tag{27}$$

Technical change is said to be factor i using, neutral or saving, depending upon whether

$$\begin{aligned} \partial S_i/\partial t &= \gamma_{it} > 0, \\ \partial S_i/\partial t &= \gamma_{it} = 0 \text{ , or} \\ \partial S_i/\partial t &= \gamma_{it} < 0. \end{aligned} \tag{28}$$

2.9 Functional Form Selection

Christensen, Jorgenson and Lau introduced the translog function in 1973 and Diewert introduced the generalized Leontief function in 1971. Since then several other flexible form functions have been introduced in the literature. Among all the flexible form functions, the translog has had the widest application.

Berndt, Darrough, and Diewert (1977) undertook a study of some of the flexible form functions and observed the

following features.

1. On theoretical grounds there were no reasons to discriminate between the translog, generalized Leontief and the generalized Cobb-Douglas. All three functions

> provide a second order differential approximation to an arbitrary twice continiously differentiable reciprocal indirect utility function which is linearly homogeneous along the ray of equal prices (Berndt, Darrough and Diewert 1977, p. 661).

2. On econometric grounds too, no reasons a priori could be found to select one form over the others, since each form involves the same independent variables and the same dependent variables and hence similar likelihood functions.

3. Given that the functions are non-nested, classical test procedures cannot be utilized to discriminate among the flexible forms.

The authors then conclude that selection is best made on a posteriori grounds by examining the parameters estimated on the basis of a priori expectations. The translog was thought to perform best given the data set employed. Yet in another paper, Appelbaum (1979) utilized a Box-Cox transformation function to generate flexible form functions and applied parametric tests to discriminate among the generalized, square-rooted quadratic and translog functions. U.S. aggregate data of the manufacturing sector utilized earlier by Berndt and Christensen (1973A) were employed. The results indicated the translog to be the least preferred.

Guilkey and Lovell (1980), utilizing Monte Carlo experiments, report that the translog models were able to represent complex production technologies reasonably well. Second single equation models were found to perform marginally better at lower computing costs than the system of equations as in the translog case.

The present study nevertheless utilizes the translog model for several reasons. As Guilkey and Lovell (1980) point out, the translog model represents complex production technologies reasonably well. Furthermore, by imposing restrictions on the parameters of the translog model, a variety of simpler production technologies can be generated. Since no systematic analysis of the production structure of the sawmills in Alberta has yet been done, the use of the translog model does provide a good starting point. Sequential hypothesis tests follow in order to select the translog model that most appropriately represents the production structure.

3. ESTIMATION TECHNIQUE AND RELATED ISSUES

3.1 Introduction

Several econometric issues are involved in estimating the translog cost function parameters and conducting various statistical tests in order to select the translog cost function that most appropiately represents the production structure of the sawmills in Alberta.

Since there are a large number of models nested within the nonhomothetic model, the most general case, an efficient method for model selection that avoids estimation of all models is desirable. The concept of ordered sequential hypotheses testing of nested models is used and provides an efficient method of accomplishing this task. Several test statistics used are used to test appropriate hypotheses.

3.2 Estimation Technique

The estimation technique is carried out in a sequential order starting from the nonhomothetic cost function. Since the estimation technique involved is the same for all translog functions which are members of the nonhomothetic model, the discussion below is confined to the nonhomothetic model only.

For estimation purposes the nonhomothetic model is rewritten in its stochastic form as

$$\ln C_t = \gamma_0 + \gamma_y \ln Y_t + \gamma_t t + \Sigma_i \gamma_i \ln P_{it} +$$
$$(1/2)\ \Sigma_i \Sigma_j \gamma_{ij} \ln P_{it} \ln P_{jt} + \Sigma_i \gamma_{iy} \ln P_{it} \ln Y_t +$$
$$\gamma_{yt} \ln Y_t\ t + \Sigma_i \gamma_{it} \ln P_{it} t + (1/2)\ \gamma_{tt} t^2 +$$
$$(1/2)\ \gamma_{yy}\ (\ln Y)^2{}_t + U_t, \tag{29}$$

for all i,j = L, K, W, M.

In equation (29), U_t is the disturbance term. Firms may not always be in a position to minimize costs due to fluctuations in input prices, labor and other inputs contracts, technical innovations and other such factors. Factor usage, however, may be assumed to tend towards optimal combinations (i.e., cost minimizing input bundles) over time. Such deviations away from the cost minimizing expansion path are assumed to be captured by the random disturbance term, U_t.

Ordinary least squares (OLS) can be utilized to estimate the parameters of the stochastic translog model. If heteroscadasticity or autocorrelation are present, the appropriate transformations can be made and generalized least squares (GLS) can be utilized to estimate the parameters.[27] Neither of these approaches, however, may

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[27]See the paper by Rockel and Buongiorno (1982) where ordinary least squares was applied to estimate the translog functions. The authors had to use the OLS because input cost share data were missing.

prove to be very efficient for a number of reasons. First, multicollinearity among the cross-product of prices cannot be ruled out and this problem of multicollinearity is likely to increase as the number of cross product terms increases. The adding up restrictions permit specification of most of the independent arguments in the cost function in terms of relative prices and help mitigate multicollinearity to a certain extent. However, multicollinearity cannot be avoided entirely (Madalla, 1977). Nevertheless, certain diagnostic tests can be conducted to test the severity of the problem.

Second, when ordinary or generalized least squares techniques are used, the information contained in the share equations is not being utilized. By not using the share equations the model is constrained to only the information contained in the cost function. Also, the degrees of freedom are reduced when share equations are not jointly utilized to estimate the parameters of the cost function (Christensen and Greene, 1976). A more desirable and efficient approach is one in which the cost function, along with the factor share equations are estimated jointly as a multivariate equation system.

The four cost share equations corresponding to the nonhomothetic cost model can be written compactly in stochastic forms as

$$S_{it} = \gamma_0 + \Sigma_{ij}\gamma_i \ln P_{it} + \gamma_{iy}\ln Y_t + \gamma_{it}t + U_{it}. \qquad (30)$$

The observed cost share equations (for all i= L, M, W, K) are assumed to be distributed stochastically along the cost minimizing expansion path. A change in input prices causes cost shares as well as total cost to change and consequently the disturbance terms in all the equations are affected. Stated differently, unsuccessful cost minimizing efforts which cause firms to deviate from the optimal input mixes are assumed to be captured by the disturbance terms.

An implicit assumption made while deriving the cost share equations from the cost function is that, in each period, there is a complete adjustment in input usage after an external price shock and no lagged adjustment in input usage occurs. Despite the implausibility of this assumption and as far as the author is aware, present estimation techniques do not permit utilization of the lagged adjustment process in the multivariate system of equations.

The above properties of the disturbance terms are generally assumed to be as follows:

1. The mean value of the disturbances is zero,

$$E(U_{it}) = 0; \tag{31}$$

2. The disturbances are assumed to have a finite variance, which however, is different for each equation,

$$E(U_{it},U'_{it}) = \sigma_{ii}\ I;\ \text{and} \tag{32}$$

3. Since the disturbance terms across a set of equations are correlated, the covariances between these disturbances are nonzero but the lagged covariances are zero

$$E(U_{it}, U'_{jt}) = \sigma_{ij}\, I. \tag{33}$$

where in the above, E is an expectations operator, U_{it} is the column vector of the disturbances, σ_{ii} are the variances, σ_{ij} are the covariances, and I is an identity matrix.[28]

The system of equations that exhibits the zero mean and non-zero variance covariances of the disturbance term was first discussed by Zellner (1962). He called it the seemingly unrelated system of equations. The method that was proposed by Zellner was to estimate the parameters of this system by deleting one share equation and then applying the GLS approach. One equation has to be deleted in order to avoid the linear dependency that exists in the system due to the adding up condition. However, this technique is sensitive to which cost share equation is deleted and hence several alternative parameter estimates can be obtained. Kmenta and Gilbert (1968) have shown that if an iterative technique is adopted with the convergence being set at a desirable level of accuracy (usually 1 percent), the

[28]Here the σ's refer to variances and should not be confused with the σ's used earlier to define substitution elasticities.

parameter estimates are insensitive to the cost share equation deleted. Second, when convergence is achieved, the parameter estimates are equivalent to the maximum likelihood estimates. This iterative technique is utilized to estimate the parameters of the model by deleting one cost share equation and setting the convergence criterion to one percent. Thus, deletion of one cost share equation takes into account the linear dependency problem and the iterative technique with the one percent convergence criterion results in estimates that are consistent and asymptotically unbiased.[29]

In the literature most authors have estimated the system of cost share equations alone. When the maintained hypothesis is other than that of nonhomotheticity, no loss results from estimating the share equations alone. However, if the maintained hypothesis is one of nonhomotheticity, then estimating the share equations alone, as argued by Christensen and Greene (1976), is unsatisfactory since the scale parameters γ_{iy} and γ_{yy}, which appear only in the cost equation, cannot be estimated when share equations alone are estimated. In this study, the entire system is utilized to estimate the paramaters and to conduct the hypotheses tests.

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[29]For more details on the seemingly unrelated system see Zellner (1962), Kmenta and Gilbert (1968), Kmenta (1971) and Taher (1983). The LSQ (non-linear least squares) available in Time Series Package (TSP) was utilized to estimate the parameters of the model.

3.3 Ordered Sequential Hypothesis Testing Of Nested Models

The purpose of this section is to explain the concept of nested models and the manner in which ordered sequential hypotheses tests are carried out.[30] The nonhomothetic translog cost function with homogeneity in input price[31] and with imposed symmetry restrictions represents the most unrestricted model which incorporates a fairly complex production technology. It is the maintained hypothesis in this thesis. All the other models (discussed in chapter 2) are deemed to be generated by imposing various restrictions on the nonhomothetic production technology in an ordered sequential manner. Hence all the translog restricted models are said to be nested within the nonhomothetic model.

The procedure to carry out the sequential hypotheses test when models are nested, as in the present case, can be explained as follows. The testing can be carried out in increasing or decreasing order of restrictiveness. For example, if one starts out by estimating the nonhomothetic model and then imposes the homothetic restriction and continues to increase the order of restrictiveness, the sequential order of restrictiveness is said to be of an increasing order. On the other hand, if the method followed is one where restrictions are sequentially removed, the case of decreasing order of restrictiveness results. One advantage of the latter approach is that the parameter

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[30]A detailed discussion of this procedure can be found in Mizon (1978) and Harvey (1981).
[31]This restriction is the direct result of production theory.

estimates from the restricted model can be utilized as starting values of lower ordered models that follow and hence computing costs are likely to be minimized.

The sequential order of hypotheses tests depends upon whether one starts out to estimate the most restricted model (Cobb-Douglas) or the most unrestricted model. Preliminary results have indicated that the model characterizing the sawmills in Alberta is more of the unrestrictive type and as a result the analysis commences by estimating the nonhomothetic model first. The hypotheses tests are carried out in increasing order of restrictiveness. The sequential hypotheses testing is then carried out by formulating a sequence of hypotheses in increasing order of restrictiveness.

Testing is conducted between a hypothesis and the one immediately preceeding it. For example, if the null hypothesis is that of nonhomotheticity which is tested against the alternative hypothesis of weak homotheticity and if the null hypothesis is rejected, the next hypothesis test is between weak homotheticity (maintained hypothesis now) and strong homotheticity. The hypothesis test is stopped whenever, between hypothesis H_i and H_{i+1}, hypothesis H_{i+1} is rejected. For example assume that the null hypothesis H_i, is the production structure characterized by the nonhomothetic translog cost function. This hypothesis is then tested against the alternative hypothesis of homotheticity (H_{i+1}). If the test reveals that the null hypothesis cannot be

rejected then the testing procedure should no longer be continued. The nonhomothetic translog cost function is then accepted as the model that most appropriately represents the structure of production under investigation.

How a variety of models can be generated by imposing restrictions in a sequential manner on the nonhomothetic translog model can be illustrated below.

Models	Restrictions
Nonhomothetic	All parameters are estimated
Nonhomothetic Strong Input Separability	Set γ_{ij} one at a time =0
Homothetic	Set all γ_{iy} =0
Nonhomothetic Hicks Neutral	Set all γ_{it} =0
Homogeneous	Set all γ_{iy} and γ_{yy} =0
Homothetic and Strong Separability	Set all γ_{iy} =0 and γ_{ij} one at atime =0
Homogeneous Strong Separabilty	Set all γ_{iy} =0 and γ_{ij} one at atime =0 and γ_{yy} =0
Homothetic Hicks Neutral	Set all γ_{iy} and γ_{it} =0
Homogeneous Hicks Neutral	Set γ_{yy} and all γ_{iy} and γ_{iy} =0
Nonhomo. and Unitary Elast. of Subs.	Set all γ_{ij} =0
Homoth. and Unitary Elast. of Subs.	Set all γ_{ij} and γ_{iy} =0
Homog. and Unitary Elast. of Subs.	Set all γ_{ij} and γ_{iy} =0 and γ_{yy} =0
Cobb Douglas	Set γ_{yy}, γ_{tt}, and all γ_{iy}, γ_{it} and γ_{ij} =0

A variety of other models can be generated in a sequential manner before the Cobb-Douglas model. Also note that weak separabilty and homotheticity tests are conducted after a relevant model is estimated.

3.4 Test Statistics

There are several different models nested within the nonhomothetic translog model as illustrated above and one model has to be selected statistically from the entire group. This section outlines two test statistics that are generally employed to carry out model selection.

The set of models generated from the nonhomothetic model involves some form of restriction(s) on the nonhomothetic model. Some of the restrictions were also seen to be nonlinear. More specifically the weak separabilty assumption involved at least one nonlinear restriction on the general model. The choice of the test statistic depends on whether restrictions are linear or nonlinear besides other criteria. Quite often, imposing nonlinear restrictions makes estimation difficult or, if possible, at costs that may not be insignificant. The likelihood ratio and the Wald test have been selected to carry out hypothesis tests in this study.

The likelihood ratio test is based on the maximum likelihood estimation. The likelihood function is a formula corresponding to the joint probability distribution of the variables of interest where the variables are assumed to be fixed but the paramaters are allowed to vary. Maximizing this function with respect to the unknown parameters gives the maximum likelihood estimates.[32]

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[32]See Kmenta (1971) and Harvey (1981) for more details.

The likelihood ratio test involves the likelihood values of an unrestricted (LV) and restricted (LVN) functions. In other words if the null hypothesis (H_0) is true, the likelihood value of the unrestricted model is expected to be large relative to the likelihood value of the restricted model. The likelihood ratio LR, given g degrees of freedom, has $\chi^2(g)$ distribution and in logarithms is defined as follows:

$$LR = 2(\log LV - \log LVN) \sim \chi^2(g). \tag{34}$$

The use of the likelihood ratio test requires the estimation of the restricted as well as the unrestricted models. Quite often the restricted models are difficult to estimate, especially when the restrictions imposed are nonlinear. The Wald test comes in handy in such a situation and requires estimation of only the unrestricted model.

The application of the Wald test involves a criterion to see how far the nonlinear restriction is away from zero. For example, when the material input is weakly separable from all the other inputs, the restriction is

$$\gamma_{im} = \gamma_i \rho_m . \tag{35}$$

where ρ_m is an unknown constant. Now, if the nonlinear restriction in equation 35 is valid, the difference between

the left and right values should be very close to zero. The null hypothesis that the restriction is valid cannot be rejected. Thus, when the difference gets larger, the nonlinear restriction has a greater chance of being rejected.[33]

The possibility exists that two or more functions can give rise to values for which the nonlinear restrictions are equal. In such a case, information on the curvature of the functions become important. The information on the curvature provides an indication as to which of the two functions is moving away faster from the value implied by the nonlinear restrictions. The information on the curvature is then utilized as weights on the Wald statistics. However, (ρ_m) is an unknown constant and is eliminated from the above equations (35) in the following manner. When material input is weakly separable from labor and wood, the weak separability restrictions (presented above in Chapter 2)[34] for i = L and j = W, are,

$$\gamma_{im} = \gamma_i \ \rho_m \text{ and } \gamma_{jm} = \gamma_j \ \rho_m \ . \tag{36}$$

Taking the ratio of the above two equations, ρ_m is eliminated and the following equation is obtained

[33]For greater exposition on the Wald and the likelihood tests see Buse (1982) and Harvey (1981).

[34]Because one share equation has to be deleted, namely the capital share equation, the coefficients of the capital variable are derived residually. Hence in the case of weak separability there is only one independent restriction.

$$\gamma_{im}/\gamma_{jm} = \gamma_i/\gamma_j \text{ or } \gamma_{im}\gamma_j - \gamma_{jm}\gamma_i = 0. \tag{37}$$

Equation 37 is the required restriction which has to be imposed in the nonhomothetic model to test for weak separability between material and all other inputs. The information on the curvature is now obtained from equation 37 by taking the partial derivative of the function with respect to all the parameters of the nonhomothetic or any pertinent function. Representing the vector of the partial derivatives as $\{\partial R/\partial\Lambda\}$, the Wald statistics (W) in the above case of weak separability can be written as in equation 38.

$$W = (\gamma_{im}\gamma_j-\gamma_{jm}\gamma_i)' \{(\partial R/\partial\Lambda)\ \Sigma^{-1}\ (\partial R/\partial\Lambda)'\}^{-1} (\gamma_{im}\gamma_j - \gamma_{jm}\gamma_i) \sim \chi(g). \tag{38}$$

The Wald statistic has a χ^2 distribution with degrees of freedom equal to the number of independent restrictions (g). In equation (38) Σ is the variance-covariance matrix of the coefficients of the unrestricted function.

3.5 Hypothesis Tests

The nonhomothetic translog model is the maintained hypothesis which is tested against several alternative hypotheses. The alternative hypotheses are generated in a sequential manner by imposing restrictions or testing for the validity of restrictions on the nonhomothetic model. The restrictions are imposed in an ascending order of

restrictiveness.

The test for weak homotheticity implies two independent nonlinear restrictions and the Wald test is utilized to test the validity of the nonlinear restrictions. Only the nonhomothetic model needs be estimated to test the weak homotheticity hypothesis. Following weak homotheticity tests, Hicks neutral technical change hypothesis is tested. This hypothesis on technical change contains three independent restrictions. Finally, the test for homogeneity contains four independent restrictions and this hypothesis is tested against the nonhomothetic hypothesis. The weak separability hypothesis does not require estimating any new equations and can be conducted from the nonhomothetic model as well. The log likelihood ratio test is utilized to test between nonhomotheticity, Hicksian neutral technical change, homotheticity and homogeneity.

The above tests constitute the first round of tests. In the event all of the models except the homogeneous one cannot be rejected, more restrictive models have to be generated until the H_i hypothesis cannot be rejected against the H_{i+1} hypothesis, given that the latter hypothesis is more restrictive. In other words the test procedure is abandoned when the maintained null hypothesis in a given round of tests cannot be rejected against the alternative hypothesis (Mizon (1978) and Harvey (1981)).

4. DIVISIA INDEXING PROCEDURE, DATA REQUIREMENTS, CONSTRUCTION AND DISCUSSION

4.1 Introduction

This chapter discusses the data set and the transformation of the raw data used to estimate the translog cost function models for Alberta sawmills. The second section outlines the Divisia indexing procedure. Also narrated are reasons for selection of this procedure and how the Divisia index is compatible with the translog functions.

The period of this study covers 23 years, starting in 1959. The definition of the unit of observation from which information on sawmills is collected by Statistics Canada was changed in 1962. In section three, Chow test is conducted to see whether any significant differences in input usage or expenditure on inputs can be discerned before and after the above noted change.[35] Finally, the last section contains discussions on the data set utilized for this study. In Chapter 7, cross-sectional data are utilized to study the scale economies in the sawmilling industry. Since Ondro and Williamson (1982) have documented the cross sectional data in detail, only a brief summary of the important points of the data are presented in Chapter 7.

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[35]The main reason for conducting this test is to be able to increase the sample size beyond 20 observations, given that the translog cost function contains 21 independent parameters. See chapter 3.

4.2 The Divisia Indexing Procedure

In order to estimate the translog cost function specified in chapter 2, input price indexes corresponding to the inputs of labor, capital, wood and materials are required.

The discrete Divisia price and quantity indexes can be defined respectively as follows:

$$\ln P_t - \ln P_{t-1} = \Sigma_i W_{it} (\ln p_{it} - \ln p_{it-1}). \qquad (39)$$

$$\ln Q_t - \ln Q_{t-1} = \Sigma_i W_{it} (\ln q_{it} - \ln q_{it-1}). \qquad (40)$$

where $W_{it} = (1/2)(S_{it}+S_{it-1})$ are time varying weights, S_i are the cost shares, P and Q denote price and quantity indexes and the p_{it}'s and q_{it}'s are the prices and the quantities of the inputs. The discrete approximation to this Divisia index is also called the Tornqvist index.[36] The above discrete index has been adopted to derive the required input price indexes for this study.

There are several reasons for selecting the Divisia indexing procedure over say, the more commonly used Laspeyres indexing procedure.

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[36]For details on the Divisia index see Allen (1975) and Diewert (1976).

Consider the Laspeyres price index (LPI) defined as;

$$\begin{aligned} LPI &= \Sigma_i\{W_{0i}(q_{1i}/q_{0i})/\ \Sigma W_{0i}\} \\ &= \Sigma\ p_{it}q_{io}\ (p_{it}/\ p_{io})\ /\ \Sigma\ p_{io}q_{io} \\ &= \Sigma\ p_{it}q_{io}/\ \Sigma\ p_{io}q_{io} \end{aligned} \tag{41}$$

The weights used are the fixed base period weights which are base period cost shares. The index gives the change in input prices for period one as compared to period zero. The Laspeyres index is easy to compute. However, certain problems exist with this procedure. First, the same base period weights are used in all periods which implies that a producer is restricted to the original expenditure pattern when base period cost shares are used as weights ($p_{io}q_{io}$). In other words, no substitution among inputs is assumed in the Laspeyres index.[37] Second, since index numbers are related to production or cost functions [see Diewert, (1976) for an extensive treatment on the issue], linear production or cost functions are implicitly assumed when Laspeyres indexing procedure is adopted.[38] No substitution, or perfect substitution, among inputs is a very restrictive assumption as has already been discussed in Chapter 2. As a result, the Laspeyres indexing procedure does not appear to be appropriate for the present purpose.

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[37]For more details see Allen (1982) and Berndt (1978).
[38]Christensen (1975), and Berndt (1977) discuss these issues at greater length.

The discrete Divisia indexing procedure is preferred. The weights utilized in the Divisia index are flexible weights [see (W_{it}) in the discrete Divisia index above]. The discrete Divisia index gives changes in total cost following changes in input prices from one period to another. Stated differently, producers are not restricted to the same expenditure pattern as implied in the Laspeyres price index but are allowed to vary the expenditures depending on how prices have changed [see Berndt (1977), Diewert (1976) and Allen (1975)]. Given prices, changes in expenditure implies producers can substitute less expensive inputs for the inputs whose prices have increased. Thus no a priori restrictions on the possibilities of input substitution are implied in the Divisia index.

Second, Diewert (1976) has demonstrated that the Divisia index corresponds to the translog cost function. This means, that since the translog functions are flexible form functions where no a priori restrictions are placed on the partial elasticities of substitution, utilization of the Divisia index is perfectly compatible with the translog function when aggregation of various inputs has to be done. As a matter of fact, Diewert (1976) has proven that the Divisia index represents exactly the linearly homogenous translog functions.[39]

Thirdly, an index number should have an economic

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[39]Also see Denny (1979) for a similiar demonstration in the case of the Tornqvist index.

meaning.[40] Berndt (1977) provides an economic interpretation of the Divisia index in the context of energy aggregation as follows:

> The percentage (logarithmic) change in the aggregate energy quantity index is a weighted average of the percentage (logarithmic) quantity changes of the component energy-types, where the weights are the time-varying 'chained' mean expenditures or cost shares (p. 247).

In an economy where prices are continuously changing and inputs are constantly being substituted due to exogenous price shocks, the appropriateness of the Divisia index is readily justifiable. The Divisia index is utilized where ever necessary in this study.

4.3 The Chow Test

The data set utilized in this study comes from various Statistics Canada sources.[41]The period covered in the study is from 1959 to 1981, 23 years. There is, however, a problem involved in using the sample data directly prior to 1962.

Prior to 1962, Statistics Canada's unit of observation, from which information on the sawmills was collected was referred to as 'sawmills reporting' and since 1962 as 'establishment'. The difference between these units can be explained as follows. Prior to 1962, if a sawmill was engaged in other production activities besides sawmilling, information under sawmills included only the latter aspects.

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[40]See for example, Allen (1975), Christensen (1975) and Berndt (1977) for more discussion on this matter.
[41] Whenever catalogue is mentioned, reference is being made to Statistics Canada catalogues unless otherwise stated.

After 1962, so long as the major activity was sawmilling, all other acivities, if any, were reported under sawmills. In the case of sawmills 'other production activities' besides sawmilling is unlikely. However, if any major differences (statistical) exist between the two observation units, these differences would be expected to be reflected in input usages or expenditures on inputs over the two samples. For example, if wood input was utilized by the sawmills for purposes other than lumber before 1962, then a difference in the nature of wood use would be expected between the two periods. This same argument would apply to the use of other inputs as well. Similar arguments would also apply in the case of expenditures on inputs.

Several important variables that are necessary to obtain the price indexes to be utilized in the translog cost function were subjected to the Chow test in order to test for any statistically significant difference in input usages between the two periods, namely, 1959-1961 and 1962-1981. The variables selected were expenditures on energy, materials and wood as well as the physical quantity of wood utilized by the sawmills. Gross capital stock (machinery and equipment, building and construction), utilized to calculate the rental value of capital services, were also subjected to the Chow test. In the case of labor, no Chow test has been conducted, since measurement of labor input in the two periods is totally different. Details on labor input is taken up below.

The Chow test is designed to test whether two subsamples belong to the same structure or to two different structures. If input usage in the two periods are different, then according to the Chow test, the regression coefficient(s) of the equations pertaining to the two subsamples are not the same statistically and the null hypothesis that input usage in the two periods is the same is rejected.

Several different models (simple linear, log linear, semilogarithmic) were fitted to the data with each of the above mentioned variables as the dependent variable (Y) and time (t) as the independent variable. In all cases, the models with the highest R^2's and significant t-value at or over 95 percent were selected and the F statistic,

$$F= (U^2/K_1)/\{(U^2{}_1 + U^2{}_2)/K_2\}, \tag{42}$$

was calculated. In the above equation U^2, $U^2{}_1$ and $U^2{}_2$ refer to the sum of squared residuals of the regression equations with all 23 observations (1959-1981), with 20 observations (1962-1981) and the 3 observations (1959-1961) respectively. K_1 (21) and K_2 (19) are the degrees of freedom.[42]

In all cases, the calculated F values were less than the tabulated values given the relevant degrees of freedom at the 95 and 99 levels of significance. Hence the null

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[42]In all cases the simple linear model (Y_i=a+ b.T) gave the best fits. Most standard text books on econometrics discuss Chow test. See for example Koutsoyiannis (1978).

hypothesis that input usages or expenditures on inputs in the two periods are not different cannot be rejected. This test therefore allows the use of data from 1959 to 1961 along with the rest of the sample.

4.4 Price Indexes And Cost Shares

4.4.1 The Price Of Labor

The data to calculate the price of labor were obtained from Statistic Canada (Catalogue number 35-204). This catalogue reports on the manhours paid to production workers and the corresponding wage bill. The total wage bill includes all supplementary benefits received by the production workers. The price of labor or the wage rate is obtained by dividing the wage bill by the manhours paid.

The manhours paid data includes overtime work. Since wage rates increase more than proportionately as overtime work increases, wage rates calculated on the basis of manhours paid are likely to be lower than wage rates calculated using manhours worked figures. For example, consider a wage rate for overtime work is of one and half times the normal wage rate. If two hours of overtime work is performed in addition to the regular hours, manhours paid is equal to three hours plus the normal hours. However, in terms of manhours worked it is two plus the normal hours worked. As a result the wage rate calculated on the basis of manhours paid can give lower wage rates than wage rates

calculated on the basis of manhours worked. However, manhours worked data for the Alberta sawmills are not available. Therefore manhours paid data are utilized.[43]

Prior to 1962, the number of production workers, instead of manhours paid, were reported by Statistics Canada. A linear trend line was fitted using 1962-1965 manhours paid and the wage bill data. Only four observations were used because the wage rates showed a sharp increase after 1965.[44]

Owners, proprietors and clerical staff or other workers have been excluded in this study since no information on hours paid or hours worked for this group exists. Furthermore, the number in this group of workers represents a very small fraction of the number of production workers. Excluding these workers is assumed not to cause any significant error. The labor cost share, however, is likely to be slightly underestimated.

No attempt has been made to account for the quality changes in labor input. Change in the quality of labor input is reflected in the change in output levels. An account of quality changes in labor input can lead to biases in the labor input series if similar quality changes are not made in the other inputs. Finally the cost of labor is calculated

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[43]Lower wage rates obtained from manhours paid data reflect lower compensation to labor and hence is less preferred. A trip to Statistics Canada in Ottawa proved futile in attempting to obtain manhours worked data for the sawmills in Alberta.

[44]Several different models were fitted and the linear model gave the best fit.

as the product of the wage rate and manhours paid. The price index of labor is calculated using the Divisia indexing procedure.

4.4.2 The Price Of Wood

Most of the sawmills in Alberta utilize only softwood logs to produce lumber output. Some sawmills use hardwood, but the volume of hardwood logs utilized as an input is insignificant relative to the volume of softwood logs. The quantity figures on hardwood are not reported by Statistics Canada for a large number of the early years. As a result, the price of wood calculated utilizes only expenditures on softwood logs and the purchased softwood log volumes.

Some sawmills have affiliated logging operations and sawlogs are transferred from these logging units to the sawmills. The transfer of logs from such units to sawmills are reported by Statistics Canada. The total volume of sawlogs as an input into the sawmills is therefore an aggregate of purchased softwood volumes, softwood transferred within logging operations and the hardwood volumes reported. The total expenditure on wood input is this wood quantity multiplied by the implicit price of wood calculated in the manner described above.[45]

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[45]Wood quantity figures and the expenditure on softwoood figures are available in Statistics Canada catalogue 35-204.

4.4.3 The Price of Materials

The material inputs component is an aggregate of energy input and a bundle of manufacturing and nonmanufacturing inputs. Statistics Canada does not report on the quantities of the manufacturing and nonmanufacturing inputs utilized by the sawmills. Also energy consumption by the sawmills prior to 1975 is not available.

To calculate the price of material input several steps are involved. First, the material inputs are divided into energy and others inputs, the latter consisting of manufacturing and nonmanufacturing inputs. Within the energy input, there are nine energy sub-types, namely, coal and coke, natural gas, gasoline, kerosene, diesel oil, light fuel oil, heavy fuel oil, liquified petroleum gases and electricity purchased. For some years, expenditures on wood and steam along with an 'others' category are also reported by Statistics Canada.

Aggregate energy consumption data for the sawmills across Canada have been utilized to calculate the price of each energy sub-type. First all energy quantities are converted to BTU units using the conversion factors reported in Table 4.1. Secondly, there are differences in the efficiency units of each energy sub-type in the production of output, with electricity being the most efficient. Thus all input BTU's are normalized relative to electricity efficiency BTU output units by utilizing the efficiency factors also reported in Table 4.1.

Table 4.1: Conversion Factors

Fuel Type	Natural Units to B.T.U. Conversion Factors	Input B.T.U. to Output B.T.U. Conversion Factors
Electricity	3.412 BTU/MKWH	1.00
Natural Gas	1030 MMBTU/MMcf	0.85
Liquified Petroleum Gas	4.095 MMBTU/Barrel	0.85
Gasoline	5.222 MMBTU/Barrel	0.20
Heavy Fuel Oil	6.2874 MMBTU/Barrel	0.87
Light Fuel	5.8275 MMBTU/Barrel	0.82
Kerosene	5.6770 MMBTU/Barrel	0.82
Diesel Oil	5.8275 MMBTU/Barrel	0.26
Coal	24.8 MMBTU/Ton	0.87

Notes: MMBTU= Millions of B.T.U.
MKWH= Thousands of KWH.
MMcf= Millions of Cubic Feet.
Source: Taher (1983).

The aggregate data on energy consumed do not contain expenditures on kerosene, diesel oil, light and heavy fuel for a number of years. Disaggregation was done by first calculating the mean shares from the most disaggreate series and then multiplying the total expenditure on the four energy sub-types by the mean shares. Expenditures on wood, steam and the 'others' category were distributed over the nine categories using the mean values as the multipliers. Energy generated by sawmills, if any, is not taken into account separately, since the costs involved to generate energy are assumed to be reflected in the other cost components reported by the sawmills. Furthermore, certain sawmills use boilers that run on wood wastes to generate energy but are not taken into account due to the lack of information. Energy consumed by sawmills is likely to be downward biased but not significantly large enough to make any appreciable change in the results given that only few sawmills have boilers. The individual energy sub-type prices were obtained by dividing the expenditure on energy by the BTU output units. For the years 1959-1961, energy prices were extrapolated. The price of energy obtained in this manner was assumed to be the price faced by the sawmills in Alberta. The quantity of energy consumed by the sawmills in Alberta was obtained by dividing the expenditure on energy borne by Alberta sawmills by the price of energy calculated.

The questionnaire sent out by Statistics Canada to the sawmills was examined to get an idea of the other components

of the material inputs. A major input in this category was wood input. Also, expenditure on materials includes expenditure on wood as well. Since separate reporting is done on wood expenditure this amount was subtracted from the expenditure on materials to obtain expenditure on other inputs.[46] The other inputs category included products such as steel wire, staples, unbleached paper kraft 1 and 2, polyethelene and film sheet, dunnage and so on.

The production and the value of shipments of some of these products were retrieved from various Statistics Canada catalogues and individual product prices were calculated. Finally the material price index was derived as a Divisia index of energy and other inputs.[47]

In the literature, the manner in which materials price indexes have been calculated differs from the approach taken in this study. The method suggested by Denny, Fuss and Waverman (1979) and Fuss (1977) appears to be the most widely used method in calculating the materials price index. The method may be described as follows. Constant dollar quantity of materials is assumed to equal constant dollar output minus constant dollar real domestic product. The

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[46]Expenditure on materials inputs reported in Statistics Canada includes the expenditure on wood input purchased by the sawmills. Since expenditure is reported separately, the expenditure on materials inputs is net of wood expenditure. The expenditure on wood discussed above includes this purchased wood expenditure plus the value of wood transferred from logging operations.

[47]The production and the value of shipments of polyethylene (cat. no. 46-217), staples (cat. no. 41-006), steel wire (cat. no. 41-006) and unbleached paper (cat. no. 36-204), energy quantity and prices were utilized to calculate the price index of material inputs.

constant dollar output is defined as value added plus cost of materials.[48] The price of materials is obtained by dividing cost of materials by constant dollar quantity of other inputs.

A major reason for not adopting the above method was due to the fact that the sawmilling sector is a very small sector of the economy and the material inputs reported for the entire economy may not be a good representation of the materials utilized by the sawmills. Secondly, the Denny, Fuss and Waverman (1979) method requires data from input-output tables, namely constant dollar real domestic product, which were not available prior to 1962.

4.4.4 The Price Of Capital

The measurement of capital as an input poses the greatest difficulty among all the inputs considered in the production process. Producers purchase other inputs such as labor, energy and materials in the market and as such the market rental values for these inputs are generally available or can be imputed. When it comes to capital, however, producers are the owners of capital and a market for capital services like other inputs does not exist. The neoclassical theory of investment treats firms as renting capital or assets from other firms or from itself to obtain capital services.[49] Capital usually comes in indivisible

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[48]If this method was adopted in this study wood expenditure would have been subtracted from the cost of materials.
[49]See Hall and Jorgenson (1967). The alternative approach taken is to treat a firm as accumulating capital assets in

amounts, depreciates over time and the stock of capital is constantly being replaced over time. The flow of capital services provided by a stock of capital which is combined with other inputs to produce output is of concern here.[50] The flow of capital services is influenced by other factors besides the stock of capital. Tax policies, life of the capital, interest rate, the rate of return, rate of depreciation of the capital and investment are some of the important variables that influence the flow of capital services.[51]

The flow of capital services reflects the rate at which a given stock of capital is combined with other inputs to produce a given level of output. Hall and Jorgenson (1966) proposed a method of imputing a value to the flow of capital services which takes into account the above mentioned variables.

The derivation of the rental value or the value of capital services following Hall and Jorgenson (1967) can be described as follows. First, a relationship between the

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[49](cont'd) order to supply a flow of capital services to itself.

[50]Capital is usually assumed to "consist of all those aids to production which have been made by", (Rowan, 1968, p.43). If this definition of capital were to be adopted then, except for labor, all other inputs touched by man falls in the category of capital. This broad distinction does not provide any insight into how factor use over time has changed due to changes in factor prices. Second if the broad category above is to be followed, the implicit assumption made is that all inputs besides labor can be aggregated into one composite group. The more desirable approach is to test if such aggregation is in fact valid. This latter approach is taken in this study.

[51]See Hall and Jorgenson (1967), and Christensen and Jorgenson (1969) for more details on capital theory.

quantity of an asset acquired and the flow of services generated has to be established. This flow of services is assumed to decline geometrically over time. The authors then establish the relationship for the service price of capital as being equal to the sum of the cost of capital, the current cost of replacement and the cost of capital loss on the value of the asset. The cost of capital is the rate of return multiplied by the lagged value of the price of the asset plus the current cost of replacement which is equal to the price of the asset multiplied by the replacement rate less the cost of capital loss which equals the net change in the price of the asset.

The authors then take into account the tax rate, profit rate, investment tax credit and the present value of depreciation allowances for tax purposes on a dollar of investment to determine the price of capital services. For each asset type the price of capital[52] services is calculated separately. Then the quantity of capital is obtained by dividing the capital stock by the price of capital services calculated. Finally the price index of capital was obtained as a Divisia index of the two service prices.[53] Statistics Canada publishes information on the

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[52]For the sawmills the two asset types are structures and buildings and machinery and equipments.

[53]For more details see Christensen and Jorgenson (1969), Hall and Jorgenson (1967) and Taher (1983). This method was utilized by Sherif (1981) while analysing the production structure of the pulp and paper industry in Canada. Also see Denny, Fuss and Waverman (1979) who have taken an approach very similar to the Hall-Jorgenson method.

capital stock for the sawmills in Alberta.[54]

The sawmill's capital stock data are reported under construction capital (CC) and machinery and equipment capital (MC) and are available both in constant (1971) and current dollar values. Furthermore, the series also contains gross and net capital stock data. The gross capital stock series in current dollars is utilized.

For the two types of capital, the service price (SP_i) is calculated using the Hall-Jorgenson formula defined as,

$$SP_i = PA_{it}(RR + D_i)\ (1-ECPTR.\ PV)/(1-ECPTR). \qquad (43)$$

for i= CC, MC and the symbols have the following meanings.

ECPTR is the effective corporate profit tax rate. This rate is obtained by dividing the tax paid by the sawmills by the profits before tax.[55]

PV is the present value of capital cost allowance generated by an investment of one dollar.[56] For the sawmills this value was calculated to be 0.01729.

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[54]Details of the method (perpetual inventory method) is reported by Statistics Canada in Fixed Capital Stocks-Manufacturing Industries at the Three Digit Level (1970 SIC): Alberta 1955-1983, which is published by the National Wealth and Capital Stock Section, Construction Division.

[55]For details see Christensen and Jorgenson (1969). The information on profits before and after taxes are available in cat. no. 61-003.

[56]See Taher (1983) for more details in calculating the present value.

PA$_{it}$ (i= CC and MC) is the implicit investment deflator. Given the depreciation rate U_i for each type of capital, the investment series can be calculated. The implicit investment deflator can then be obtained by dividing the investment in current dollars by investment in constant dollars.[57]

RR is the real rate of return taken as the Mcleod, Young and Weir average of 10 industrial bond rates and was obtained from Data Research Incorporated.

D_i is the constant depreciation value taken to be equal to .065 and .092 for construction and machinery capital respectively. These figures were obtained from Bigsby (1983).

Finally the aggregate capital service price P_k is calculated as a Divisia price index of the construction and machinery capital.

4.4.5 Output

The real gross output of sawmills is considered as the output variable entering as an argument of the translog cost function. The real gross output is obtained by dividing the value of shipments from the sawmills and includes the sales

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[57]See for example, a macro economics textbook such as Rowan (1968) for the definition and the method of deriving the index.

of chips by the industry selling price index.[58]

4.4.6 Total Cost and Cost Shares

The total cost (C) is defined as the sum of the expenditures on the four inputs,

$$C = P L + P K + P W + P M . \tag{44}$$

where the P_i's are input prices corresponding to the inputs of labor (L), capital (K), wood (W) and materials (M). The input cost shares are the ratios of the individual expenditures to total cost.

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[58]The industry selling price index is obtained from cat. no. 62-528.

5. DISCUSSION OF TIME SERIES RESULTS

5.1 Introduction

The translog models (Chapter 2) and the estimation technique and related issues (Chapter 3) are utilized together with the data (Chapter 4) to select the most appropriate representation of the production structure of sawmills. The economic analysis of the production structure follows.

The input price indexes and the cost shares calculated are discussed next. Then, several translog models are estimated and the appropriate test statistics are conducted to select the most representative model. The own and cross-price demand elasticities and elasticities of substitution are examined along with the technological change biases in input use and scale effects that may have occured in the sawmills over time.

5.2 Changes in Factor Prices and Cost Shares

Factor price indexes and the cost shares calculated are depicted respectively in Figures 5.1 and 5.2 and in Appendix Tables 10.1 and 10.2. The price of labor or the wage rate shows the most significant increase. The price of labor index reported in the figure is based on manhours paid and not manhours worked. Wage rates based on manhours paid are likely to be lower than those based on hours worked.[59]

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[59]The total wage bill is the same in both cases, but since the number of manhours paid when overtime worked is involved

However, the wage rates calculated reflect the actual cost borne by the sawmills on average to hire a unit of production labor when overtime work is required in the sawmills.

The other input prices show less increase relative to the price of labor, although the prices of other inputs too have increased over the years. The price index of wood appears to have declined for a few years (1960 till 1962) and then increased again. Only by 1969 did the price index of wood reach its pre-1960 value.

The price of capital registers the most variation. The price of capital as calculated in this study depends, among other things, on the bond rate and the tax variable which have not remained constant over time. The large variation in the price of capital, therefore may reflect the variations in the bond rate and the tax variable. The price of materials, which also includes the price of energy shows a steady increase (see figure 5.1). After 1973 the material price index recorded a very large increase, reflecting partially the energy price increase as a result of the international oil embargo by the OPEC nations. Prior to 1973 the price index of material, however, reveals only a moderate increase.

Despite the large increment in the price of labor the sawmills have been able to keep labor costs fairly constant (see Figure 5.2), which they have achieved by employing less

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[59](cont'd) exceeds the hours worked, wage rates based on hours worked exceed the wage rates based on manhours paid.

Figure 5.1 Trends in Input Price Indexes: Alberta Sawmills

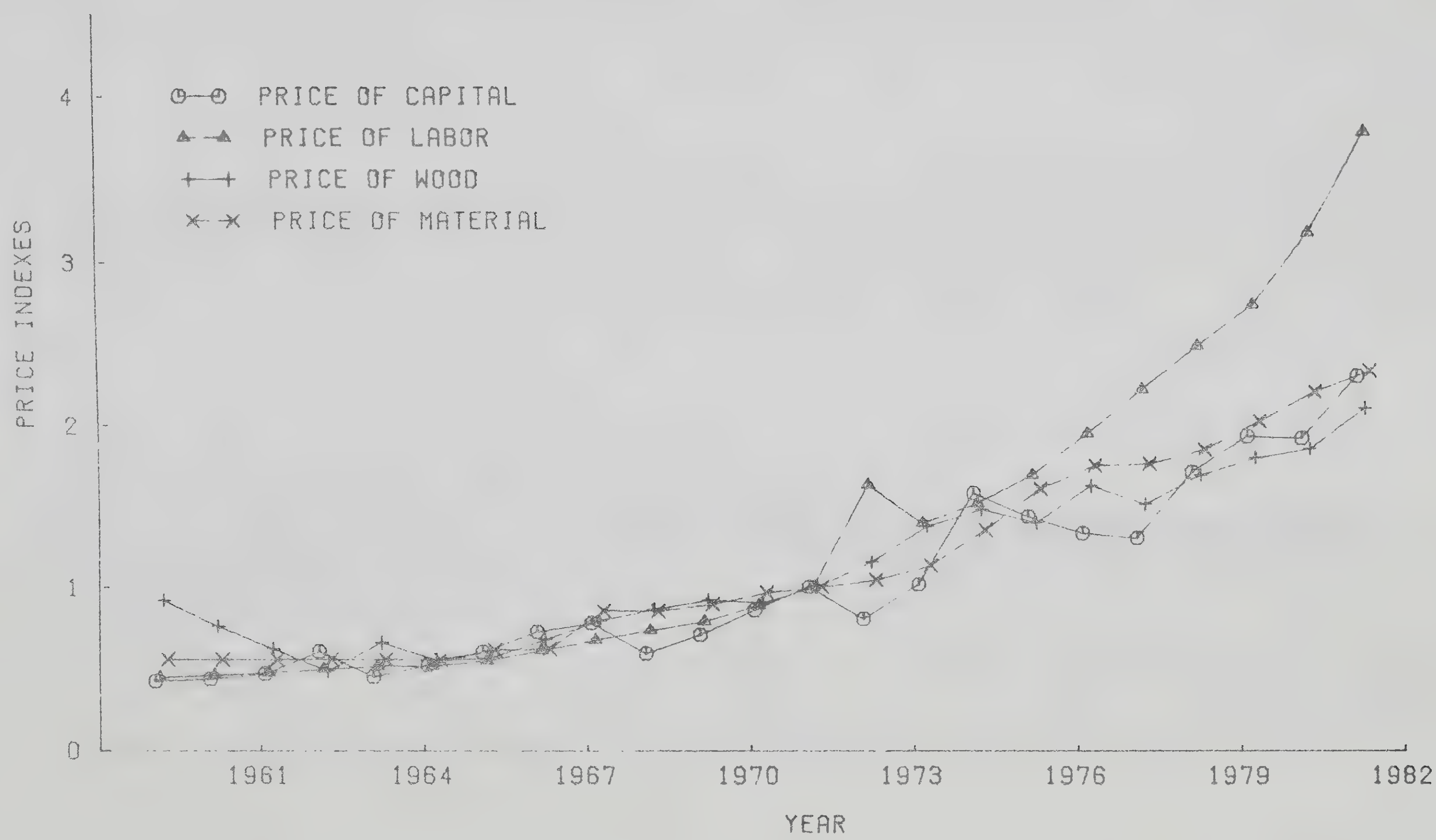
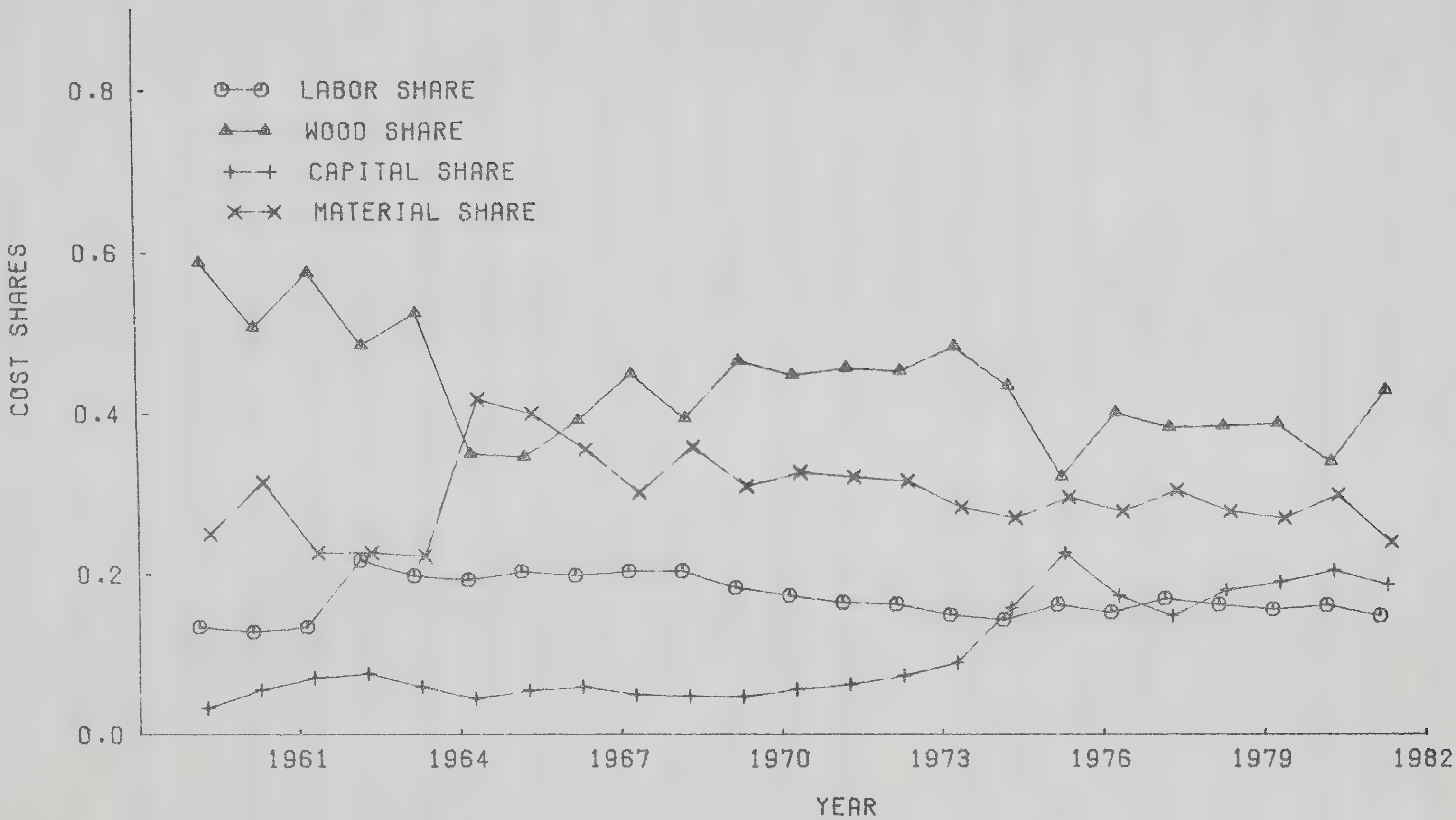


Figure 5.2 Trends in Input Cost Shares: Alberta Sawmills



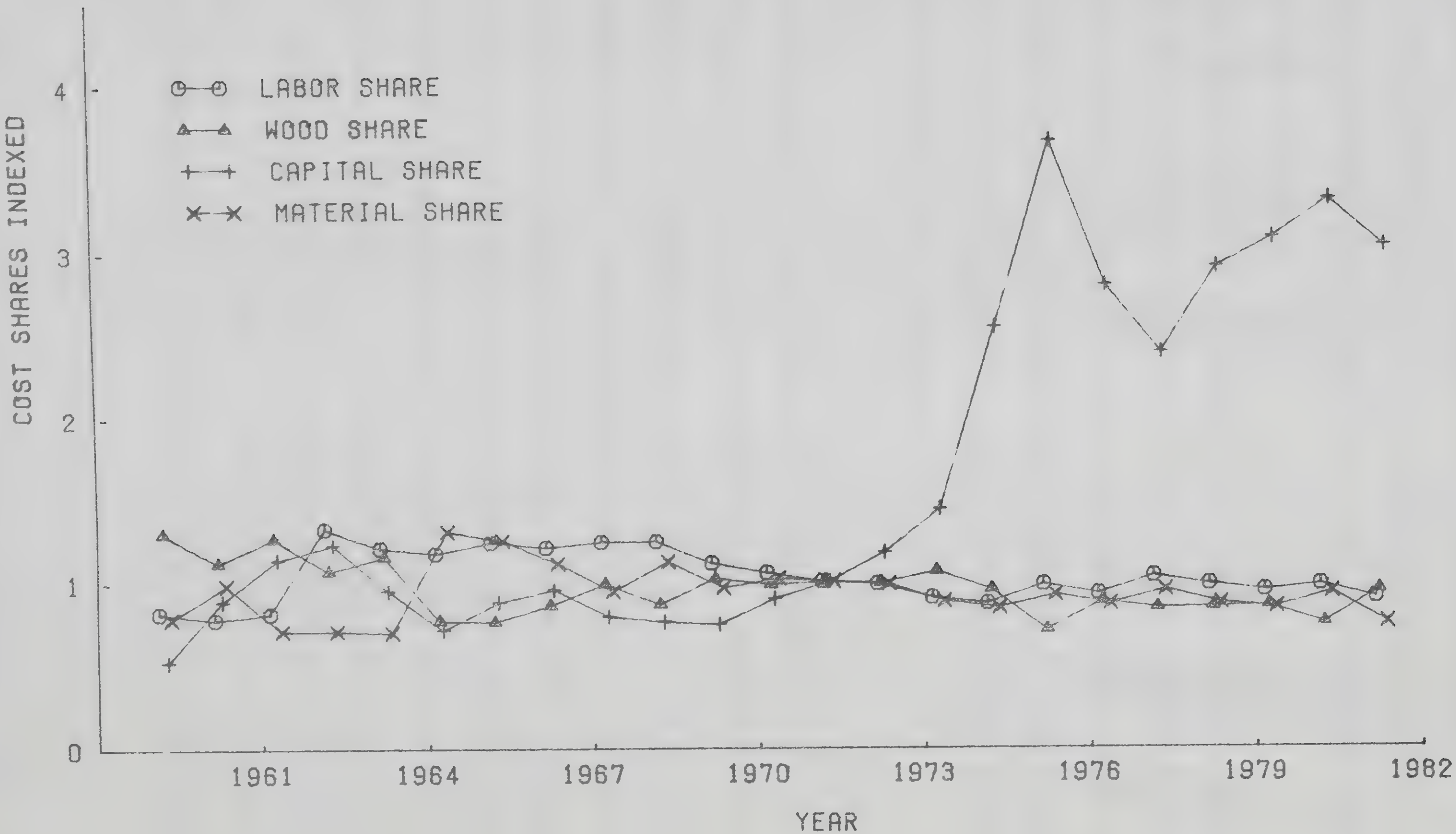
labor over time. The same result appears to hold largely for wood and material cost shares as well. The capital share on the other hand has increased fairly significantly over the 23 year period and also registers the greatest variation (see Figure 5.3).

5.3 Production Characteristics in the Sawmilling Industry

This section is devoted to a description of the procedure used to select the cost function that most appropriately represents the production technology in the sawmills of Alberta.

The nonhomothetic model is the maintained hypothesis and is tested against several alternative hypotheses. Following the nonhomothetic model (model A) the least restrictive model is the homothetic model (model B) with three independent restrictions (all γ_{iy} =0). The tests for weak and strong homotheticity involve nonlinear restrictions and separate models need not be estimated, but can be tested utilizing the parameter estimates of the nonhomothetic model. However, if another model is selected instead of the nonhomothetic model, the weak and strong homotheticity tests must be conducted using the model that is selected. Hicksian neutral technical change (model D) involves four independent restrictions (γ_{tt} and all γ_{it} = 0) which are different from the homothetic restrictions. This later model is also estimated. The homogeneity restrictions (γ_{yy} = 0 and all γ_{iy} = 0) are imposed on Model A and the model (model C) is

Figure 5.3 Trends in Indexed Input Cost Shares: Alberta Sawmills



reestimated. All remaining (dozen or so) restricted models can be generated by imposing more restrictions on these four models. The Cobb-Douglas cost function is a special case of the nonhomothetic model when all second order parameters are restricted to zero.

The above models were each estimated as a multivariate regression system consisting of the cost equation and the cost share equations, with the capital share equation deleted for reasons discussed in chapter 3. The homogeneity in factor price and symmetry restrictions were imposed in all the models prior to estimation.

5.3.1 Selection of the Production Structure

Whether or not the estimated cost functions are well behaved has to be determined prior to model selection. A well behaved cost function has to satisfy the monotonocity and convexity in input prices conditions. The monotonicity condition is satisfied if all fitted cost share equations are positive. The convexity condition is satisfied if the bordered Hessian matrix is seminegative definite. Both of these conditions were satisfied by all the estimated cost functions at all 23 data points.[60] Hence the estimated cost functions are said to be well behaved in the neoclassical sense.

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[60]The time series processor (TSP) package was used for estimating the multivariate system. The convergence criterion was set at one percent. In all cases convergence was achieved in about 30 to 45 iterations.

The estimated coefficients of the models presented in Table 5.1 reveal that, in the nonhomothetic model (Model A), three of the four homothetic paramaters (γ_{iy}'s) are significantly different from zero. Also an examination of the biased technical change parameters (γ_{it}'s) in the same table indicates that at least three of these paramaters are significant.

Table 5.2 presents results of the likelihood ratio and Wald tests conducted to test the null hypothesis of nonhomotheticity against the several alternative hypotheses.[61] The calculated χ^2 values lead to the conclusion that all of the restricted models are rejected at both the five and one percent levels of significance.[62]

The above tests therefore lead to the conclusion that in order to study the production structure of the sawmills in Alberta, the nonhomothetic model is most appropriate. Following Mizon (1978) and Harvey (1981), the need to estimate the remaining nested models does not arise. The concept of ordered sequential hypotheses testing of nested

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[61]Note that in the preceeding paragraph only the t-values of the parameters (γ_{iy}'s and γ_{it}'s) were examined to see if they were significantly different from zero. The likelihood ratio test is a more general test that takes into account the entire function.

[62]In the models γ_{yy} and γ_{tt} are both set to zero. In an earlier run these paramaters were insignificant and second the variables associated with these parameters namely, $(\ln Y)^2$ and $(t)^2$ were believed to be a major source of multicollinearity. Auxuliary regressions conducted by first deleting $(\ln Y)^2$ and then t^2 improved significantly the results of the model and hence the suspicion that the two variables were contributing fairly significantly to multicollinearity was confirmed. As a result, the above two variables have been deleted in all estimations.

Table 5.1: Estimated Coefficients of Different Translog Cost Functions: Alberta Sawmills 1959-1981

	Model A	Model B-C	Model D	Model E	Model F
γ_o	-1.7703*	-1.2033*	-1.3546*	..	-1.6250*
γ_y	0.3854*	0.2000*	0.2367*	..	0.3023*
γ_t	0.0536*	-0.0922**	-0.0805**	..	0.0385*
γ_l	0.0453	0.1934*	0.1663*	0.0570	0.1684*
γ_m	0.0254	0.3350*	0.2980*	0.0031	0.2968*
γ_w	0.5895*	0.4501*	0.5628*	0.6091*	0.4337*
γ_k	0.3397*	0.0215	-0.0271	0.3379*	0.1011*
γ_{ly}	0.0536*	..	0.0032	0.0533*	..
γ_{my}	0.1133*	..	-0.0001	0.1304	..
γ_{wy}	-0.0506	..	-0.0392	-0.0688	..
γ_{ky}	-0.1164*	..	0.0361	-0.1149*	..
γ_{lt}	-0.0054*	-0.0081	..	-0.0063*	..
γ_{mt}	-0.01260*	0.0040	..	-0.1149*	..
γ_{wt}	0.0027	-0.0014	..	0.0067	..
γ_{kt}	0.01528*	0.0072*	..	0.0149*	..
γ_{ll}	-0.0340	-0.0159	-0.0432	-0.0081	..
γ_{lm}	0.1211*	0.0994**	0.0310	0.149*	..
γ_{lw}	-0.1122*	0.0900*	0.0310	0.1070*	..
γ_{lk}	0.0251	0.0066	0.0572**	0.0281	..
γ_{mw}	-0.0565**	-0.0432	-0.0134*	-0.1789**	..
γ_{mm}	-0.0123	-0.0138	-0.0432	0.0972	..
γ_{mk}	-0.0523	-0.0423	-0.0251	-0.0254	..
γ_{wk}	0.0124	-0.0220	-0.1082**	-0.0045	..
γ_{ww}	0.1564*	0.1552*	0.1662*	0.3104*	..
γ_{kk}	0.0148	0.0577	0.0760	0.0018	..
γ_{yt}	0.0316*	0.0333*	0.0291*	..	..
D.W.	2.1365	1.6649	1.8067	..	1.32
LLV	188.350	177.920	172.276	159.656	145.750

Notes: 1. Models A,B,C,D,E and F refer respectively to the nonhomothetic, homothetic, homogeneous, Hicks neutral technical change, implied nonhomothetic and the Cobb-Douglas cost functions.
The * and ** refer to significance at 1 and 5 percent respectively. The t-values of the estimates are provided in Table 5.1A.
LLV and D.W. refer to the log of likelihood values and Durbin Watson statistic respectively. Also note that Model E does not contain the cost function and hence the Durbin Watson statistics is not reported.

Table 5.1A :t-Values of Estimated Coefficients of Different Translog Cost Functions: Alberta Sawmills 1959-1981

	Model A	Model B-C	Model D	Model E	Model F
γ_o	5.15	4.78	4.64*	..	6.29
γy	3.54	2.67	2.80	..	3.32
γt	2.57	2.31	2.03	..	5.59
γl	1.09	15.20	3.48	1.39	31.88
γm	0.19	12.77	3.10	0.02	28.39
γw	4.16	17.58	6.72	4.53	29.97
γk	4.96	1.17	0.34	4.91	7.88
γly	3.75	..	0.23	3.80	..
γmy	2.42	..	0.01	2.99	..
γwy	1.01	..	1.70	1.45	..
γky	4.79	..	1.59	4.70	..
γlt	3.95	1.59	..	4.55	..
γmt	3.40	1.94	..	4.14	..
γwt	0.68	0.71	..	1.69	..
γkt	7.54	4.87	..	7.27	..
γll	1.37	0.57	1.71	0.30	..
γlm	3.96	2.74	0.81	3.16	..
γlw	4.76	3.26	1.78	5.30	..
γlk	1.35	0.25	1.90	1.45	..
γmw	2.04	1.53	0.48	2.48	..
γmm	0.29	0.32	0.88	0.99	..
γmk	1.53	1.02	0.49	0.54	..
γwk	0.38	0.58	2.68	0.14	..
γww	3.76	3.82	4.22	4.17	..
γkk	...	...	...	...	...
γyt	3.40	3.46	3.09	..	..

Note: There are m (4) separate equations (the cost function and 3 share independent equations) and n (23) observations on each of the n equations. This gives N= n.m (92) observations in all. Each share equation has 6 variables and the nonhomothetic model will yield estimates of 18 paramaters. Add to this the 19 parameters of the cost function as well. The resulting degrees of freedom, thus is 55. (See Johnston, 1984, p. 339).

models (Chapter 3) suggets that a hypothesis test be abandoned when, between two hypotheses H_i and H_{i+1}, the H_{i+1} th hypothesis is rejected, given that the H_{i+1} hypothesis is more restrictive than the H_i hypothesis. Hence, the results discussed in the remaining portion of this chapter are confined to the nonhomothetic model only unless otherwise mentioned.

The case of global separability is not satisfied. A test for global separabilty is equivalent to testing whether or not the Cobb-Douglas cost function appropriately represents the production structure of the sawmills. The estimated coefficients of the Cobb-Douglas cost function (Model F) are reported in Table 5.1. The log likelihood ratio test results are reported in Table 5.2. The calculated χ^2 values exceeds the tabulated χ^2. The specification of a Cobb-Douglas function as an appropriate representation of the production structure characterising the sawmills is therefore rejected.

Based on the likelihood ratio test, the nonhomothetic translog cost function cannot be rejected as appropriately representing the production structure of the sawmilling industry. However, within a nonhomothetic production structure input usage can exhibit weak or strong separability relationships. The results are presented in Table 5.2. The highly significant value of the Wald statistics leads to the rejection of the weak separability hypothesis. Strong (or linear) separabilty for an input

Table 5.2: Test Statistics for Model Selection: Alberta Sawmills

Models	No. of Restrs.	X^2(1percent)	X^2(5percent)	Cal X^2
Model B-C	4	13.28	9.49	20.86
Model D	4	13.28	9.49	32.15
Model F	4	13.28	9.49	672.59
Model G	2	9.21	5.99	572.67
Model H	3	11.35	7.82	15.86
Model I	3	11.35	7.82	12.54
Model J	3	11.35	7.82	13.67
Model K	13	27.69	22.36	85.26

Notes: The models refer to the following:
Models B and C to homotheticity and homogeneity;
Model D to Hick's neutral technical change;
Model F to weak separabity in inputs;
Model G to weak homotheticity;
Model H, I, and J to strong input separabilty of labor, material and wood respectively; and
Model K is the Cobb-Douglas function or the case of global separabilty in inputs;
The Wald test statistics is applied to models F and G and to the remaining models the likelihood ratio test was applied.

requires that all the cross-product terms, in which the input in question is to be strongly separable from all other inputs, be set equal to zero. By setting $\gamma_{ij} = 0$ (i,j= L, W, M) in the nonhomothetic model three new models are generated. These models are estimated and the likelihood ratio test is conducted. Again the results point to the conclusion that strong separability for each input cannot be accepted.

Several conclusions emerge from the acceptance of the nonhomothetic production structure. First, changes in output affect input demand functions. Second, the sawmills are not characterized by constant returns to scale and are also constrained by factor prices in altering their production operations. Third, the sawmills exhibit biased technical change in input usage. Fourth, there appears to be a significant degree of factor substitution in the sawmills in response to external price shocks. Also, sawmills do not exhibit weak or strong separability in input use. Stated differently, each input usage is neither weakly nor strongly independent of other input usage. Hence the cost function for the sawmills in Alberta cannot be specified in terms of subcost functions. Also global separability is rejected. Finally, both the weak and strong homotheticity assumptions are rejected.

5.3.2 A Note on the Nonhomothetic Model

In some studies nonhomothetic models have been estimated without the inclusion of the cost function in the multivariate system on the grounds that the number of observations was inadequate.[63] When only share equations are estimated, the γ_y and γ_{yy} (scale) as well as other parameters, cannot be evaluated. To see how sensitive the nonhomothetic model is to the exclusion of the cost equation, the 'implied' nonhomothetic model without the cost equation is estimated utilizing only the cost share equations. Except for one paramater (γ_{mm}) which had a different sign and a coefficient that was statistically insignificant, other parameters estimated were consistently similar in both models. The log likelihood value dropped from 187.81 (nonhomothetic) to 159.49 ('implied' nonhomothetic), which is to be expected when variables are dropped from models. The paramaters estimated and the statistical significance of the implied nonhomothetic model are more consistent with those of the nonhomothetic model than are those estimates of the other models (see Table 5.1).[64] Christensen and Greene (1976) argued that the optimal procedure to evaluate a nonhomothetic cost structure is to estimate jointly the cost equation along with the cost share equations as a multivariate regression system. The

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[63]For example, see Binswanger (1974B), and Taher (1983), where the authors have made use of cost share equations alone to estimate the parameters of a nonhomothetic model.
[64]Note that the log likelihood value of a restricted model can, at most, be equal to that of the unrestricted model but never greater.

above results point out that estimating the share equations alone in the absence of sufficient sample points can provide a reasonable approximation of the nonhomothetic model in this case.

5.4 Comparative Static Results

5.4.1 Own and Cross Price Elasticities

Input demand and substitution elasticities were computed for all the 23 years. A sample of the elasticity estimates is provided in Tables 5.3 and 5.4. Also, the mean values of the elasticities are reported in these tables. Note that the values for 1981 are terminal estimates for the sample.[65]

All own-price elasticities computed have the expected negative sign. The results presented in Table 5.3 reveal that these own-price elasticities have remained fairly stable over the 23 years covered by this study. The own-price elasticity of demand for labor is the most elastic among the set of elasticities.

The own-price demand elasticities of capital and material are similar in magnitude, even though capital elasticity has been slightly greater than that of materials in most of the years. The change in the values of these

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[65]The standard errors of the elasticities are not reported. Generally, when the standard errors are calculated the assumption is made that the cost shares are nonstochastic and this is not true. Secondly, standard errors of the elasticities are generally not reported.

elasticities over the years have been similiar. In at least 17 out of the 23 years covered by this study, whenever the own-price elasticity of material inputs changed, the own-price elasticity of capital also records a similiar change suggesting a complementarity relationship between these two inputs.

The own-price elasticity of demand for wood is the least elastic among the four inputs considered in all the 23 years covered by this study. This elasticity has remained fairly stable over the years. The relatively low value of all three elasticities over the years is to be expected, given the 'basic good' nature of this input in the sawmilling industry.

A sample of the cross-price demand elasticities is reported in Table 5.4. Notice that the cross-price elasticities are not symmetric unlike the substitution elasticities which are reported below. The first column should be read as the effect of a change in the demand for factor i, when the price of factor j changes and all other prices and output are held constant. These cross-price elasticities are related to the substitution elasticities as given by the relationship, $\eta_{ij}=S_j\sigma_{ij}$. In other words, cross-price elasticities are proportional to the substitution elasticities. The partial substitution elasticities provide better measures of the relationships between inputs when relative prices change and hence attention is directed to the elasticities of substitution.

Table 5.3: Own Price Demand Elasticities: Alberta Sawmills

	Labor	Wood	Material	Capital
Mean	-1.0381	-0.1965	-0.7452	-0.6915
1961	-1.1243	-0.1540	-0.8297	-0.7168
1966	-0.9750	-0.2091	-0.6808	-0.6877
1971	-1.0439	-0.2013	-0.7187	-0.6968
1976	-1.0734	-0.2091	-0.7674	-0.7419
1981	-1.0844	-0.2067	-0.8124	-0.7341

However, prior to discussing the substitution elasticities, it is interesting to compare these results with those of others.

A difficulty arises at this point in that the results of other studies are not strictly comparable with this one. First, other studies have covered a different sector of the economy (e.g., the manufacturing sector). Second, inputs utilized differ in number and types. Third, some studies have employed functional specifications different from ours. The data base generally differs among studies, if not in terms of the data sources, at least in terms of the assumptions made concerning various issues.

In a study conducted by Sherif (1981) of the pulp and paper industry in Canada, the price elasticities estimated were much lower than our estimates.[66] Stier (1980B) reports on a study conducted by Mckillop in which the own-price wood demand elasticity was found to be as high as -3.2. In the same paper Stier reports that subsequent studies have estimated elasticities between 0 and -.5, which compare with results herein. In a recently completed study, Taher (1983) estimated labor and capital own-demand elasticities to be -.3846 and -.2950 and -.2192 and -.8163 for the wood and furniture industries respectively in Canada.

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[66] Sherif's (1981) study utilized a nonhomothetic nonneutral Hicksian translog specification. Her study utilized energy as a separate input instead of material. The other inputs were the same.

Table 5.4: Cross Price Demand Elasticities : Alberta Sawmills

	Mean	1961	1966	1971	1976	1981
Capital-Labor	0.5191	0.4934	0.6257	0.5720	0.2974	0.2817
Wood-Labor	-0.0970	-0.0633	-0.0902	-0.0828	-0.1291	-0.1152
Material-Labor	0.5873	0.6709	0.5395	0.5425	0.5886	0.6537
Labor-Capital	0.2531	0.2588	0.1859	0.2145	0.3373	0.3567
Wood-Capital	0.1301	0.0910	0.0902	0.0886	0.2025	0.2150
Material-Capital	-0.0799	-0.1630	-0.0892	-0.1021	-0.0172	-0.0326
Labor-Material	1.0328	1.1394	0.9678	1.0593	1.0775	1.0631
Capital-Material	-0.4340	-0.5278	-0.5396	-0.5317	-0.0278	-0.0419
Wood-Material	0.1634	0.1263	0.2090	0.1955	0.1357	0.1069
Labor-Wood	-0.2479	-0.2739	-0.1784	-0.2300	-0.3414	-0.3353
Capital-Wood	0.6064	0.7512	0.6016	0.6564	0.4722	0.4943
Material-Wood	0.2379	0.3218	0.2305	0.2782	0.1960	0.1913

5.4.2 Partial Elasticities of Substitution and Complementarity

The substitution-complementarity relationship between inputs are displayed in Table 5.5. From the results presented in Table 5.5, the elasticities appeared to have changed more sharply between 1971 and 1976 than in other periods. This period (1971-1976) contains the years when global energy prices increased in an unprecendented manner. The sharp changes in the elasticties therefore, reflects the change in the relative price of inputs and the response of the producers to such changes. Similiar trends may also be observed in the cross-price elasticities reported earlier in Table 5.4.

Labor displays a substitution relationship with capital and material inputs and a complmentarity relationship with the wood input. The substitution relationship between labor and capital, however, has declined steadily over the years. The larger values of these substitution elasticities in the early years may reflect a time when the sawmills in Alberta began introducing new capital which appears to have displaced fairly large amounts of labor.[67] The results further indicate that the degree of substitution has declined gradually over the years. These results appear to be fairly consistent with the findings reported by Stier (1982A) and Greber and White (1982). Their estimates are,

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[67]Stier (1982A) and Greber and White (1982) indicated that in the U.S. lumber industry the displacement of labor has been mostly of unskilled type.

Table 5.5: Partial Elasticities of Substitution: Alberta Sawmills

	Labor Capital	Labor Material	Labor Wood	Capital Material	Capital Wood	Material Wood
Mean	3.0941	3.5654	-0.5987	-1.4735	1.3945	0.5442
1961	3.7252	5.0657	-0.4779	-2.3464	1.3106	0.5613
1966	3.1716	2.7346	-0.4571	-1.5245	1.5414	0.5906
1971	3.4932	3.3130	-0.5054	-1.6629	1.4423	0.6114
1976	1.9659	3.8904	-0.8531	-0.1002	1.1801	0.4898
1981	1.9166	4.4478	-0.7838	-0.1754	1.1553	0.4471

however, lower.[68] The gradual decline in the substitution relation between labor and capital over the years may reflect an increasing difficulty associated with replacement of skilled labor by capital. Sherif (1981) estimated 0.15 as the substitution elasticity between labor and capital. For the U.S. manufacturing industry, Berndt and Wood (1975) estimated this elasticity to be in the order of 1.01. Stier (1980B) estimated a value of 0.194 for the lumber industry. Denny and Fuss (1977) found a complementarity relationship between labor and capital in their study of the Canadian manufacturing industry. Furthermore Taher's (1983) estimates for the wood and furniture industry are respectively 0.0301 and 0.1126.

When the demand for wood increases, the demand for labor also increases in the sawmilling industry as may be observed by examining the complementarity relationship between these two inputs in Table 5.5. In the short run, when output is expanded, sawmills can do so by increasing wood input requirements. In order to increase wood requirements, more labor appears to be hired by the sawmills given that labor is relatively more flexible than capital in the short run.

Labor and material inputs display a substitution relationship, which is fairly strong and fairly stable over time. Given the broad aggregate nature of the material

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[68]Stier (1982A,1982B) also found capital and labor to be substitutes in the U.S. lumber industry, but his estimates are much lower than the present ones.

inputs, the substitution relationship between this input and all the other inputs has to be interpreted with caution. Energy is part of the material inputs. In some studies, labor and energy have been found to be substitutes in the manufacturing sector.[69] But the energy component in the material inputs is fairly small (8 to 10 percent) and the substitution relationship exhibited cannot be attributed to energy alone. Besides energy input, the other components of the material input consists of a group of heterogenous products such as staples, steel wire, paper and other inputs. The substitution relationship displayed above may be the result of the efficiency gains in the use of the material inputs, which have led to a decline in labor use. However, further interpretation of this elasticity is not warranted without more information on the material inputs. Furthermore, the cost share of material inputs is second only to the cost share of wood, making material a fairly important input in the sawmills.

A complementarity relationship between capital and material was found, but it has been declining over the years. The sharp increase in energy prices appears to have motivated the sawmills to utilize relatively more energy efficient capital. The complementarity strength in 1981 was only about eight percent of that in 1959. Capital displays a substitution relationship with wood. New technology has made it possible for better wood utilization. Notice, however,

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[69]For example, see Berndt and Wood (1975), Sherif (1981), and Taher (1983).

that the capital-wood substitution relationship also shows a gradual decline over the years.

Finally wood and material are substitutes. If a large part of the material inputs consists of packaging materials, then a complementarity relationship between wood and materials would be expected. However, at this point, interpretation of the elasticity becomes difficult without more information on the material inputs. The elasticity has remained fairly constant over the years.

The results presented in Table 5.5 indicate that the mean elasticity estimates may not always describe the processes of factor substitution that takes place, especially in the temporal framework. The trend appears to be more important. For example, the mean value of the labor-capital substitution relationship is 3.09 and the terminal value of this elasticity is about half that value. By examining the trend and the terminal values, a better picture of the process of factor substitution emerges. Moreover, the terminal values may prove to be better values than the mean value for prediction purposes, especially if the elasticities reveal a definite trend.

5.4.3 Scale Effects and Technical Change Bias in Input Usage

When output changes, input usage can vary in different ways. More of some inputs may be required and less of other inputs may be demanded. The results of Model A in Table 5.1 indicate that three out of the four scale parameters (γ_{iy},

homotheticity parameters) are significantly different from zero. Particular note is made of the fact that the increase in the level of output in the sawmills tends to increase the demand for labor and materials but decreases the demand for capital. The negative association between capital and output may reflect some form of overcapitalization in the sawmills given that the sawmills have become a very capital intensive industry in recent years. It could also reflect under-utilization of capital as well, since no account of the capacity utilization rate for the sawmills has been taken into account in the study. A reason for not taking capacity utilization into account is the fact that even when capital is not being used, the owner must still bear a cost. Accounting for capacity utilization can underestimate the rental price of capital. The positive association between labor and output may reflect the need for skilled labor to operate the new technology when the demand for wood output increases. Wood input apparently has no significant scale effect.

The results of the hypothesis tests indicated that biased technical change in the sawmilling industry could not be rejected. The technical change (γ_{it}) paramater, for which time is used as a proxy, is a poor indicator of technical change. Technical change is very unlikely to occur at a constant rate as specified in the model, but rather in spurts. The exact measurement of technical change cannot be

provided from the model, but only its direction.[70] Examination of the γ_{it} coefficient of Model A in Table 5.1 reveals that technical change in the sawmilling industry has been labor and material saving and capital using. All three coefficients are significant. The labor saving technical change reflects numerous technical changes that have taken place in the sawmilling industry over the last 23 years.

The elasticity of substitution between labor and capital is high and means that the isoquant in the labor-capital axis is fairly steep but convex to the origin. At the same time the labor saving and and capital using technical change biases suggest that over time the isoquant has shifted more towards the capital axis and away from the labor axis. These results also point to the fact that in the sawmills, the biased technical change along with the substitution relationship, have resulted in relatively greater savings of labor and at the same time increased the use of capital. Greber and White (1982), found that technical change in the U.S. lumber and wood industry was more responsible for the decreasing use of labor and an increasing use of capital over time than the substitution relationship between these two inputs. Alberta sawmills started introducing new technology during the early sixties which replaced labor gradually. Also wage rates have been rising faster than the price of capital which has led to further displacement of labor by capital.

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[70]For greater details on this issue see Lopez (1980), Binswanger (1974A,1974B) and Petersen and Hayami (1976).

Labor and wood are complements, but the value is fairly low. Also a fairly strong labor saving technical change bias was observed. If technical change was wood using, the above elasticity would perhaps have been smaller. The results suggest that employment in sawmills is likely to increase only if there is an increase in the demand for wood inputs. Increases in wage rates not only reduce employment in the industry but appear to reduce wood utilization as well. Capital using technical change, however, does not appear to preclude the possibility of more efficient utilization of wood.

Wood utilization is likely to increase if material prices increase. When material prices increase, ceterus parbus, material use decreases through (1) the substitution of wood for materials and (2) the technical change bias in material use. Finally, the capital using technical change bias, wood neutral technical change bias and the substitution relationship exhibited between wood and capital appear to have enhanced greater use of capital.

5.5 Summary

A detailed economic analysis of the Alberta sawmills reveals that the nonhomothetic translog cost function represents appropriately the production structure of these sawmills. There appears to be significant degree of factor substitution in the sawmilling industry. Technical change bias in input usage has been nonneutral, except for wood

which was neutral. Also global, weak, and strong separabilities among inputs are not present in the sawmills.

6. TECHNICAL INEFFICIENCY IN THE SAWMILLS

6.1 Introduction

The translog function discussed in Chapter 2 and reported in Chapter 5 is an 'average-practice' cost function or simply an average cost function which captures the average production technology characterizing the sawmills. Strictly speaking the technology characterized by the estimated cost function does not in any way reflect the minimum cost to produce output by the sawmills unless all sawmills are equally efficient in production. Productive efficiency means the ability of a production organization, such as a sawmill, to produce a well-specified output at minimun cost. In order to have knowledge of the minimum cost required to produce output, the cost (production) frontier for a given state of technology has to be estimated.

In specifying the stochastic translog cost function and the associated share equations in Chapter 3, an assumption was made that the error terms were normally distributed with mean zero and a constant variance. Thus the estimated cost function has to be an average cost function with some observed data points lying below or above the fitted function and still others lying on the estimated function itself. It is in this sense that the estimated cost function is referred to as an average practice cost function or simply an average cost function.

If a cost function can be estimated such that it passes only through the coordinates representing the minimum cost points, then the estimated cost function gives the minimum cost of producing output for a given state of technology. Such a cost function is referred to as a cost frontier or an envelope cost function or the 'best practise' function .

Many firms in an industry are likely to have cost levels that exceed the cost associated with the cost frontier.[71] Also no firms can have costs that are below the cost frontier locus. Thus the cost frontier can be utilized as a reference point in order to discern how observed costs deviate from minimum costs for any given technology. Also deviations beyond random effects of observed costs from minimum costs can be utilized to gain an insight into technical and allocative inefficiencies of the firms.

The present chapter is aimed at highlighting some of the above issues for the sawmills in Alberta. Section two is devoted to an explanation of the means by which technical and allocative inefficiencies or efficiencies can be estimated. First, the work pioneered by Farrell in 1957 is briefly discussed. Section three is devoted to a discussion of the stochastic frontier cost function. Section four utilizes the results of section three to derive sawmill technical inefficiency.

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[71] Deviations from the minimum costs can be due to factors that are within the control of the firms as well as factors that are beyond the control of the firms.

6.2 Measurement of Technical and Allocative Efficiency

In 1957, Farrell presented a method for measuring productive efficiency as well as technical and allocative efficiencies independently. The standard of efficiency utilized by Farrell was the frontier isoquant (or the linear homogenous production function). Deviations away from the frontier isoquant were identified by Farrell as the result of technical and allocative inefficiencies.[72] Farrell also recognized the possibility of decisions that affect allocative (technical) efficiency and have ramifications on technical (allocative) efficiency.[73]

Farrell's procedure for measuring technical and allocative efficiency can be outlined as follows. Assume a linear homogeneous production process which transforms two inputs X_1 and X_2 into an output Y. The standard of this production process is set by the unit isoquant (II') depicted in Figure 6.1. Farrell defined technical efficiency as the ratio OB/OA and allocative efficiency as the ratio OC/OB. The overall productive efficiency is the product of two inefficiencies, namely OC/OA.

A point such as A in the Figure 6.1 is an inefficient

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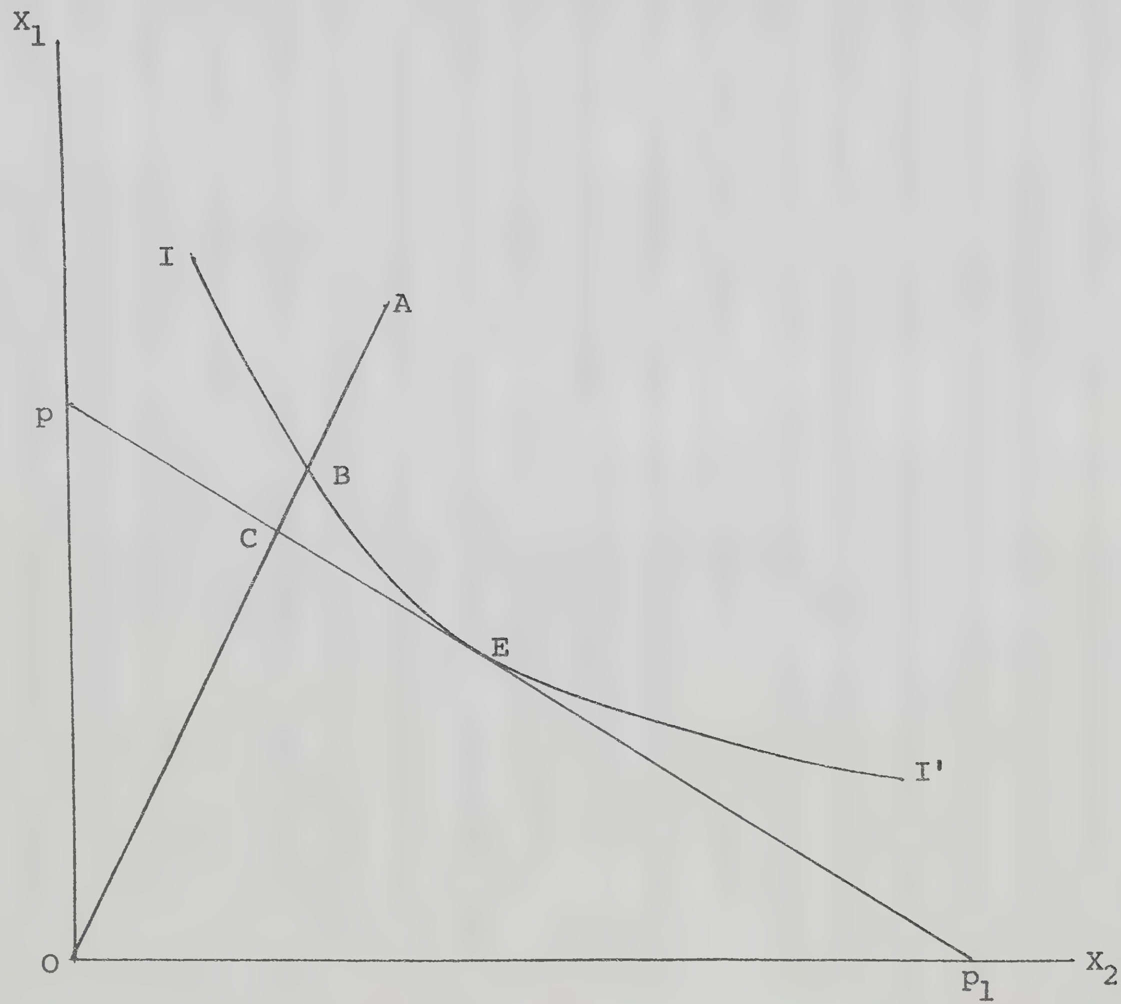
[72]Note that Farrell does not assume random factors such as wheather, pests and so on which are beyond the control of the production organization, as causes of inefficiency. In recent work, however, economists have been able to incorporate random factors as well in the stochastic function framework along with both technical and allocative inefficiencies. These later issues are dealt with below.

[73]See Farrell(1957) for more details. Farrell's original work can also be found in other studies such as Kopp (1981) and Kopp and Diewert (1982).

Figure 6.1 Farrell's Measure of Technical and Allocative Efficiencies



production plan.[74] Point C is on the isocost line PP'. PP', the locus of allocatively efficient input ratios, represents minimum costs at given competitive prices. A point such as B on the frontier represents a technically efficient point for a given level of output. E is the only point that is technically and allocatively efficient. The ratio OB/OA measures the input use at an inefficient point relative to the efficient point. Also the ratio OC/OB measures input use at B on the isocost line relative to the cost minimizing input point E. In terms of costs these efficiencies would simply be the cost ratios at the points A, B, C, and E.[75]

Farrell's approach has several shortcomings. First the assumption of a unit isoquant is limiting.[76] Second, the method precludes the possibility of exogenous factors causing deviations from the cost minimizing expansion path as well as from the frontier. Since the pioneering work of Farrell, interest in frontier functions has increased only lately despite some early work by Aigner and Chu (1968) and Timmer (1971), among others.

Aigner and Chu relaxed the assumption of a linear homogeneous production function and utilized a linear programming technique to estimate a production frontier. Since a production frontier is the locus of points that represent maximum output, the observed output points have to

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[74]In Farrell's analysis output has to be held constant and efficiencies are measured in terms of factor proportions.
[75]For more details on measuring technical and allocative efficiencies in terms of a cost function see Kopp and Diewert (1982).
[76]This assumption has already been dealt with in chapter 2.

lie on or below the frontier. Define the production function

$$Y_t = f(X_t, \beta), \tag{47}$$

where Y_t is output and (X_t) are input vectors and (β) is a vector of unknown parameters. Aigner and Chu (1968) formulated the linear programmming problem:

$$\text{Minimize } \{ Y_i - f(X_i; \beta)\}, \tag{48}$$

subject to

$$\Sigma\{Y_i \leq f(X_i; \beta)\}. \tag{49}$$

If the square of the objective function in equation 48 is taken, the problem becomes a quadratic programming problem. Notice that the set of linear restrictions clearly satisfies the requirement that observed points have to lie on or below the frontier. The estimated coefficients from the programming problem give the paramaters of the (deterministic) production frontier.[77] A major drawback with the use of programming technique in deriving a frontier function is that inferential results cannot be obtained since neither the standard errors nor the t-ratios are provided. Second, linear programming problems are extremely

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[77]Frontier functions derived from programming techniques are generally referred to as determinisitic frontiers in the literature. See Forsund, Lovell and Schmidt (1980).

sensitive to outliers [see Aigner, Lovell and Schmidt (1977)]. [78]

Recently, there have been significant developments in the area of stochastic frontiers and their estimation. Building on the earlier works, Schmidt (1976), Aigner, Lovell and Schmidt (1977), and Schmidt and Lovell (1979) have formulated stochastic frontiers and also provided methods for estimating allocative and technical efficiencies. The following section outlines the concept of a stochastic frontier and the estimation technique.

6.3 Formulation and Estimation of a Stochastic Frontier

A stochastic cost frontier can be defined as

$$C_t = C(X_{it}, Y_t, \gamma) * \exp.\epsilon_t, \tag{50}$$

where ϵ_t is a random disturbance term assumed to be distributed with mean zero and a constant variance, (σ^2), γ is a vector of parameters and the variables are as defined earlier. The random term ϵ_t is further assumed to be composed of two independent error terms, that is

$$\epsilon_t = v_t + u_t. \tag{51}$$

With two error terms, the model (eq. 50) becomes a composed

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[78]For applications of the linear and quadratic programming approaches to estimate frontier functions see Aigner and Chu (1968), Timmer (1971) and McLemore, Whipple and Spielman (1983).

error model. The first error, v_t, is assumed to have the normality properties of zero mean and $v\sigma^2$ variance. This error term v_t is assumed to capture all effects outside the control of the sawmills. The error component, u_t, on the other hand is assumed to capture errors arising out of technical inefficiencies and has a mean of zero with a variance truncated at zero from below. This truncated variance $|u\sigma^2|$ means that u_t can have only positive values, thus making all observed cost points lie on or above the stochastic frontier. Note that, even if u_t is zero, the frontier given by equation 50 above still becomes a stochastic frontier.

In specifying the composed error model, the assumption is made that firms are assumed to be allocatively efficient.[79] The measure of technical inefficiency is given by

$$\exp.(u_t) = C_t / \{C(X_{it}, Y_t, \gamma) \exp.(v_t)\}. \qquad (52)$$

When $\exp.(u_t)$ is equal to one, firms are said to be 100 percent technically and allocatively efficient.[80]

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[79] The assumption of allocative efficiency can also be relaxed and tested. However, for the purpose of this thesis due to computing problems the assumption will not be relaxed. Relaxing this assumption to actually measure allocative inefficiency requires adding one more error component and the set of factor demand equations. The nonlinear optimization problem becomes fairly complex.

[80] Note that allocative efficiency is assumed a priori. When u_t is assumed to be truncated from above and all the other properties are restored, which therefore, in the case of a production frontier, makes all observed points lie on or below the frontier function. See Aigner, Lovell and Schmidt

The method for estimating the parameters of the stochastic cost frontier, and hence technical inefficiency, involves defining the log likelihood function for the system and solving for the parameters through the maximum likelihood estimation techniques. The log likelihood function for this particular case is

$$\ln L\{C_t \mid \gamma,\lambda,\sigma\} = -N/2\ \ln(2\pi)\ -N/2\ \ln\sigma^2\ -1/2\sigma^2\ \Sigma_t\epsilon_t^2\ +$$
$$\Sigma_t \ln\{\ 1-\ F'(\epsilon_t\lambda/\sigma)\}\ \text{for } t=1...N\ . \qquad (53)$$

where N is the sample size, σ^2 is the variance of ϵ_t, λ is the ratio of the variance of the error components appearing in equation (51), and F is the standard normal distribution function corresponding to a standard density function f evaluated at (ϵ_t).[81]

The variance of v_t measures the variation in the cost due to factors that are exogeneous to the sawmills. The variance of u_t on the other hand measures the variation in cost due to factors that are within the control of the sawmills. The mean value of this error component is a measure for technical inefficiency. The term (λ) therefore provides a measure of the relative variation in cost caused by factors that are both within and without the control of the sawmills. When (λ) is large, the variance of u_t is large (or the other variance can be small) indicating that the

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[80](cont'd) (1977), Schmidt and Lovell (1979) and Bagi (1984) for greater details.
[81]See Aigner, Lovell and Schmidt (1977) and Schmidt and Lovell (1979) for further details.

cost variation (frontier vs. observed) is due more to technical inefficiency than pure random factors. Conversely, when (λ) is small, cost variations are due more to random factors beyond the control of the sawmills than to technical inefficiency.

In equation 52 the measurement for technical efficiency or inefficiency is defined but its measurement requires the value of the error v_t which is not available. The mean value of u_t, which is of interest, has been shown to be equal to

$$E(u_t|\epsilon_t) = \sigma'\{(f'(\epsilon_t\lambda/\sigma)(1 - F'(\epsilon_t\lambda/\sigma) - (\epsilon_t\lambda)/\sigma\}. \qquad (54)$$

It defines the conditional mean of u_t, given (ϵ_t), the normal distribution of v_t and the truncated normal distribution of u_t.[82] In equation 54, f' is the value of the normal density function, F' is the value of the normal distribution function and σ' is the ratio of the product of the variances of u and v to the sum of the same two variances. The ' (primes) indicate that the functions are evaluated at ϵ_t. The estimated value of ϵ_t is utilized in equation 54.

For estimation of technical efficiency, the Cobb-Douglas cost function is utilized.[83] A fortan program

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[82]For more details see Bagi (1984) on the conditional mean of u_t.
[83]A nonlinear numerical optimization process is involved in estimating the model in eq. 53. Cost and time constraints prohibited going beyond the Cobb-Douglas cost function and relaxing the assumption of allocative efficiency. Dr. R. Kopp of Resources for the Future, who frequently publishes in this area of technical and allocative efficiency

combined with BMDP package was utilized to estimate the maximum likelihood estimates of the stochastic frontier.

6.4 Discussion of Results

From a theoretical point of view, there appears to be no reason why the stochastic frontier has to be the same as the average function. Even though the more desirable approach would be to utilize a flexible form function and select the most representative model, as was done in Chapter 5, it could not be applied here. Several factors are involved. Nonlinear optimization with truncated disturbances has appeared in the literature only during the past few years thus making its application fairly limited. Second, nonlinear optimization problems are extremely complex. Without considerable experience, seeking results can be very expensive and time consuming. Therefore the Cobb-Douglas function is selected and estimated using the time series data.

Table 6.1 presents estimates of the stochastic and average cost functions. The estimated coefficients are different from each other. The intercept term of the stochastic frontier is slightly lower than that of the

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[83](cont'd) measurements, suggested by telephone that it is best to start with a Cobb-Douglas cost function given the inexperience of the author in handling nonlinear optimization algorithms. The author's short experience in this matter strongly supports the suggestion given by Dr. Kopp. Furthermore, because of the problems encountered in the optimization process, the allocative inefficiency topic had to be abandoned.

Table 6.1: OLS and Maximum Likelihood Estimates: Alberta Sawmills

Coeff.	OLS	MLE
Constant	-4.9421 (0.3934)	-6.2129 (0.0478)
Labor/Capital*	0.7383 (0.2353)	1.7000 (0.0018)
Material/Capital*	-0.4148(0.3039)	-0.2220 (0.0340)
Wood/Capital*	0.5237(0.2248)	1.010 (0.0288)
Output	0.5806(0.1116)	0.9750 (0.1141)
Scale Eco.		1.0256
Sigma Square		0.31
Sig. Sq. Star		0.0466
Lambda		2.1000
Ratio of Var(u/v)		0.8152
Av. Tech. Eff.		0.60
Av. Tech. Ineff.		0.40

* These are coefficients of price ratios.
Standard Errors in parentheses
The OLS estimates here differ from the results presented earlier in Table 5.1. Here the time variable is not included in the model and hence the differences in the two results.

average cost function.[84] The technically more efficient sawmills (as reflected by the stochastic frontier cost function) may have a lower level of fixed costs than the less efficient ones (which is assumed to be reflected in the average cost function). This interpretation carries the assumption that the intercept captures some of the fixed cost components. Furthermore, a neutral shift of the stochastic frontier from the average function would imply that the coefficients of both functions would be the same except for the intercept.[85] The estimates presented in the table show that, besides the intercept, some coefficients also differ in magnitude indicating that the shift of the stochastic frontier is nonneutral.

λ was interpreted earlier to reflect the variation in observed cost due to technical inefficiency and random factors. When λ is greater than one, the variation in observed costs away from the frontier cost is mainly due to technical inefficiency.[86] The value of λ is greater than one, clearly implying that in the sawmills technical inefficiency has contributed to cost levels that are greater

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[84]Schmidt and Lovell's (1979) intercept estimates of the two functions are similiar.

[85]See for example, the paper by Schmidt and Lovell (1979). Also see Kontos and Young (1983) where the the Cobb-Douglas production function is first estimated and the intercept is adjusted until some of the observed production points are equal to the fitted points. The method involved requires the scaling of the intercept term such that the some of the errors become zero. An approach of this nature implictly assumes that the stochastic frontier is simply a neutral shift of the average function.

[86]The interpretation of these results are can be found in Schmidt and Lovell (1979) and Bagi (1984).

than the frontier costs. Random factors which are not within the control of the sawmills have less to do with observed costs being greater than frontier costs. Therefore, the factors which reduce average production costs in the sawmills appear to be largely within their control.

A second interesting point that emerges from the result concerns the discrepancy between observed and frontier costs. The term γ captures this discrepency and its value is 0.8152. This value suggests that over 80 percent of observed costs, which are greater than frontier costs, can be attributed to technical inefficiency alone. The remaining 19 percent can be attributed to the random factors which are beyond the sawmill's control.

The fact that technical inefficiency has led on average to higher production costs implies that the sawmills have scope to reduce their average production costs. The sawmills must have some unexploited scale economies. The estimated return to scale is 1.0256 at the frontier.

Inefficiency in the sawmills can exist for various reasons. Sawmills are constantly undergoing technical change. At any point in time in the sawmilling industry or across sawmills at a point in time, differences in technology is likely to exist, with some sawmills having relatively more recent technology than others. Such differences are likely to result in the type of technical inefficiency being discussed. After an initial investment in new assets takes place, a portion of sawmills is not likely

to replace old assets each year dispite newer technology coming onto the market. At any point in time any sawmill can be efficient within its existing production structure, but when compared with another sawmill that has acquired newer technology, the sawmill with older technology will appear to be relatively technically inefficient.

6.5 Summary

A stochastic cost frontier is the best practice cost function for a given production process. Such a function corresponds to the theoretical concept of minimum cost. The result indicate that technical inefficiency in the sawmilling industry averages about 40 percent. Also, about 80 percent of the variation between minimum and observed cost is due to technical inefficiency alone. The remaining 20 percent appears to be due to random factors that are beyond the control of the sawmills.

7. ECONOMIES OF SCALE IN THE SAWMILLING INDUSTRY

7.1 Introduction

Economies of scale are best studied using cross-sectional data. Cross-sectional data reflect the fact that all firms in the sample have access to the same production technology. Economies of scale estimated from time series data do not truly distinguish cost reductions due to technological changes from the portion of cost reduced by scale economies (Christensen and Greene 1976).

Before scale economies are estimated, the process of model selection as conducted in chapter 5 must be performed. The results of three models (namely, the nonhomothetic, the homothetic, and the homogeneous models) are presented in Table 7.1. The results of the likelihood ratio tests conducted are presented in Table 7.2. The nonhomothetic model is selected for the cross-sectional sample.[87]

7.2 The Cross-Sectional Data

A panel of cross-sectional data was obtained from a survey conducted by the Canadian Forestry Service, Northern Forest Research Centre in 1978. The sample covered 83 sawmills in Alberta and is a good representation of all but the very smallest part-time operations.[88]

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[87]The discussions in chapter 2 apply equally well to the cross-sectional model by simplying setting $t = 0$ in the translog models. The estimation technique discussed in chapter 3 also applies to the cross-sectional translog model.

[88]For more information see Williamson (1983), Williamson and

Table 7.1: Estimated Coefficients of Different Translog Cost Functions: Cross Sectional Results

Coeffs.	Model A	Model B	Model C
γ	-6.5130*	-5.4210*	-4.3756*
γy	1.282*	1.1001*	0.9482*
γl	0.2210	-0.4870	-0.0419
γk	0.0999	0.1553	0.1506
γw	0.4050	0.7648*	0.7622*
γe	0.2741*	0.1285*	0.1291*
γyy	-0.0125	-0.0106	..
γyl	-0.0307*	..	..
γyk	0.0241*	..	..
γyw	0.0233*	..	..
γye	-0.0166	..	..
γll	0.1480*	0.1349*	0.1348*
γlw	-0.1383*	-0.1440*	-0.1451*
γlk	-0.0165	0.0094	0.0106
γle	0.0068	-0.0003	-0.0003
γkk	-0.0330	0.0315	0.0313
γkw	-0.0370*	-0.0142	-0.0155
γke	0.0205**	-0.0267**	-0.0263
γww	0.1740*	0.1862*	0.1886*
γwe	0.0014	-0.028*	-0.0280*
γee	-0.0287**	0.0549*	0.0546*

Note: * and ** refer to and 5 percent level of significance respectively.
Model A ,B and C are respectively the nonhomothetic, homothetic and the homogeneous models.

Table 7.1A: t- values of Coefficients of Different Translog Cost Functions: Cross Sectional Results

Coeffs.	Model A	Model B	Model C
γ	4,32	3.52	5.94
γy	6.66	5.68	7.05
γl	1.30	0.30	0.25
γk	0.59	0.84	0.82
γw	3.28	7.50	7.41
γe	5.05	2.34	2.35
γyy	0.94	0.78	..
γyl	4.43	..	..
γyk	3.72	..	..
γyw	4.03	..	..
γye	6.05	..	..
γll	7.75	6.59	6.60
γlw	0.93	0.48	0.54
γlk	12.05	12.29	12.24
γle	0.95	0.03	0.04
γkk	1.42	1.20	1.19
γkw	2.46	0.95	1.04
γke	1.79	2.01	1.99
γww	11.46	12.05	12.22
γwe	0.18	3.26	3.27
γee	1.91	2.81	2.81

Table 7.2: Test Statistics For Model Selection: Alberta Sawmills

Models	Model B	Model C
No. of Restrictions	3	3
X^2 5percent	7.815	9.488
X^2 1percent	11.345	13.277

The cross-sectional sample is not compatible with the time series information that has been used in this study . The present sample does not contain materials as a separate input. Energy input is provided instead. Energy input reported was first converted into gigajoules (metric equivalent of BTU's) and the price of energy was obtained by dividing the energy bill by the quantity in gigajoules.

Labor was measured in man-months and the price of labor was obtained by dividing the wage bill by the number of man-months employed. The price of wood was the price of delivered sawlogs at the mill gate. The price of capital is the amount paid for a dollar of capital services and was obtained by dividing the sum of depreciation and total annual maintenance expenditure by the total replacement value of the capital stock.[89] Output of the sawmills was measured in foot board measure (fbm). Finally the total cost is the sum of expenditure on each input.

Theoretically speaking, there is no reason for the relative prices of inputs in a competitive market to vary among sawmills. In reality, however, such variation can occur. Labor is not perfectly mobile and insitutional constraints can cause wages to differ among sawmills in different regions. Also, the quality of labor among sawmills is not homogeneous which is another reason why the wage can vary among sawmills if such quality differences in labor are

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[88](cont'd) Phillips (1983), and Ondro and Williamson (1982).
[89]This method of calculating the price of capital has been suggested by Varian (1978).

present. In more capital intensive sawmills, the type of labor required to operate automated and computerized equipment will command a higher wage rate than in sawmills that rely on less sophisticated equipment. Ondro and Williamson (1982) documented the fact that the quality and quantity and type of capital differs fairly significantly across the sawmill. Older capital is likely to incur higher maintenance costs and provide a reason for variation in the price of capital. The cost of transporting wood can cause variations in the price of wood at the mill. Also, small sawmills utilize isolated timber tracts which are not economical for larger sawmills, and such timber stands may be selling at lower prices. Furthermore, larger sawmills can incur lower wood costs than smaller sawmills due to economies of scale that may result in transporting wood volumes in bulk. If larger sawmills have affiliated logging operations, economies of scale which are likely to be present in such vertical integration can result in relativey lower wood costs. Differences in the observed prices of energy can be the result of delivery costs of some types of energy such as coal, diesel and other energy types which need to be transported from different locations.[90]

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[90]For studies that have utilized cross-sectional data to study production structures see Christensen and Greene (1976), Griliches and Ringstad (1971) and Sidhu (1974) to cite a few.

7.3 Economies of Scale and the Average Variable Cost

The economies of scale is defined as one minus the elasticity of cost with respect to output. When cross-sectional data are utilized the scale economies (SE) in percentage terms for a nonhomothetic translog cost function are defined as

$$SE = 1 - \{\gamma_y + \gamma_{yy} \ln Y + \Sigma_i \gamma_{iy} \ln P_i\}. \qquad (45)$$

Before the scale economies are estimated, hypothesis tests were conducted to identify the most representative model from the set of models nested in the nonhomothetic translog models in a manner similar to that described in Chapter 3.[91] The nonhomothetic model was selected (see Table 7.1 , 7.1A and Table 7.2 for the coefficient estimates, their t-values and the log likelihood ratio test results).[92] Using the relationship defined above, the economies of scale has been estimated for each firm at the observed level of output and input prices.

All 83 sawmills appear to have positive scale economies, but only 45 of these estimates were statistically significant (i.e., only 45 estimates have standard errors that are equal to or less than half the estimated value of

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[91]The levels of output between the smallest and the largest sawmills were very large therefore suggesting heteroscadasticity. The null hypothesis of homoscadasticity could not be rejected at the one percent level of siginificance when the Goldfield-Quandt test was applied to test for heteroscadasticity.

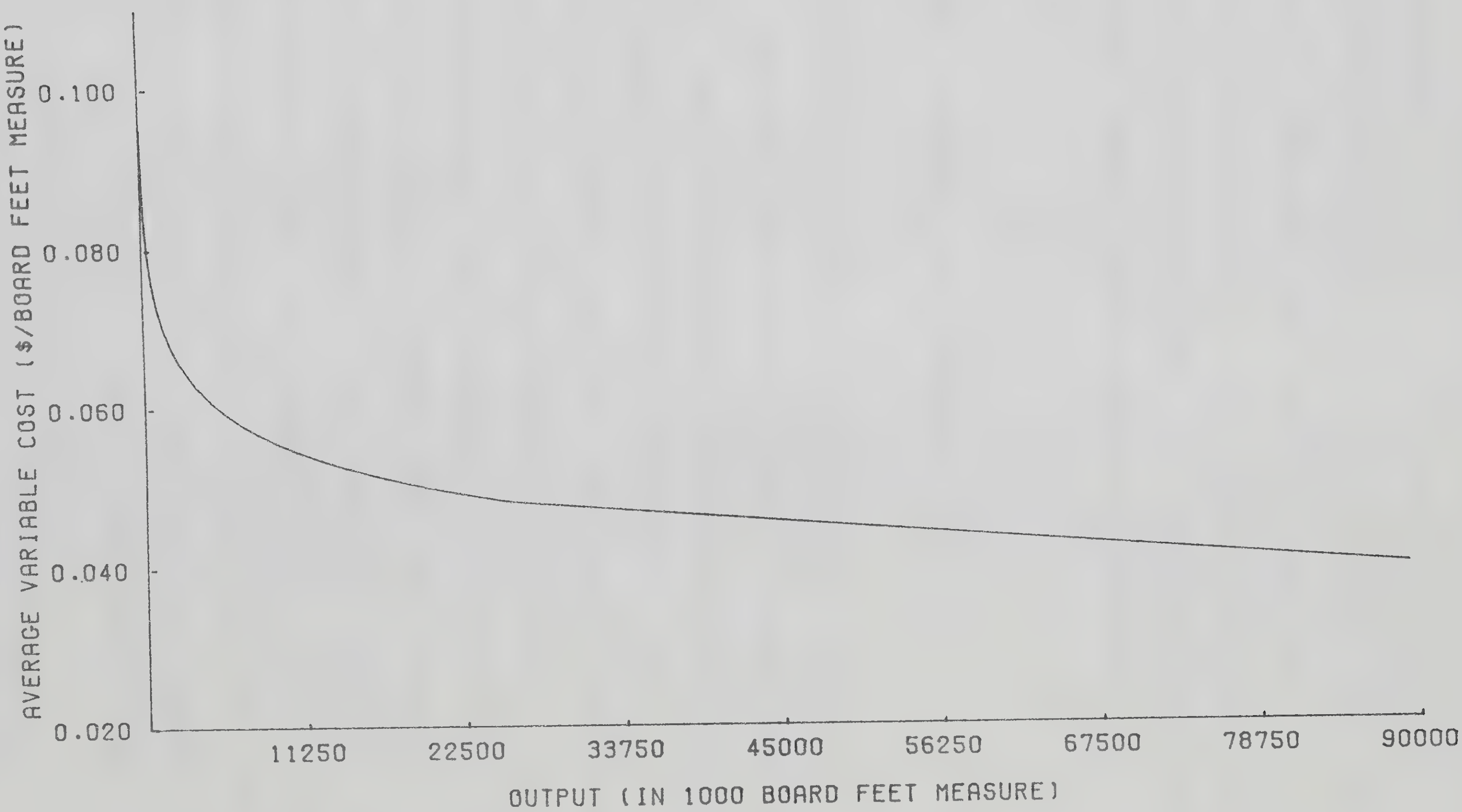
[92]Unlike in the time series model multicollineariety problems were not severe in the cross sectional models.

the scale economies). For all 83 sawmills, the estimated SE appear to increase with the level of output and vary within the upper bound of 0.17 and a lower bound of 0.05. For the 45 sawmills, the SE vary between the same previous upper bound and a lower bound of 0.10.

The average variable cost curve (AVC) for all sawmills can be derived by evaluating the cost function when all factor prices are held constant at their mean values. The AVC curve was found to be relatively flat and was declining throughout (see Figure 7.1) thus indicating that some sawmills in the industry have not yet fully exhausted their scale economies. It appears therefore that to increase efficiency in the sawmills under existing technology, firms have little choice but to increase output far in excess of existing levels. However, encouraging such an expansionary policy may not be very desirable from an equity point of view since such a policy may force many of the smaller sawmills out of the industry because of the huge capital investments that would be required for large scale expansion. Also the industry may ultimately become a monopoly. The income and unemployment problems that are likely to result if the smaller sawmills are eliminated may not be desirable given the high rates of unemployment already existing in the province and the fairly large number of smaller sawmills that exist.

Somewhat greater precision can be given to the point estimates of the scale economies by defining an acceptance

Figure 7.1 Average Variable Cost Curve: Alberta Sawmills



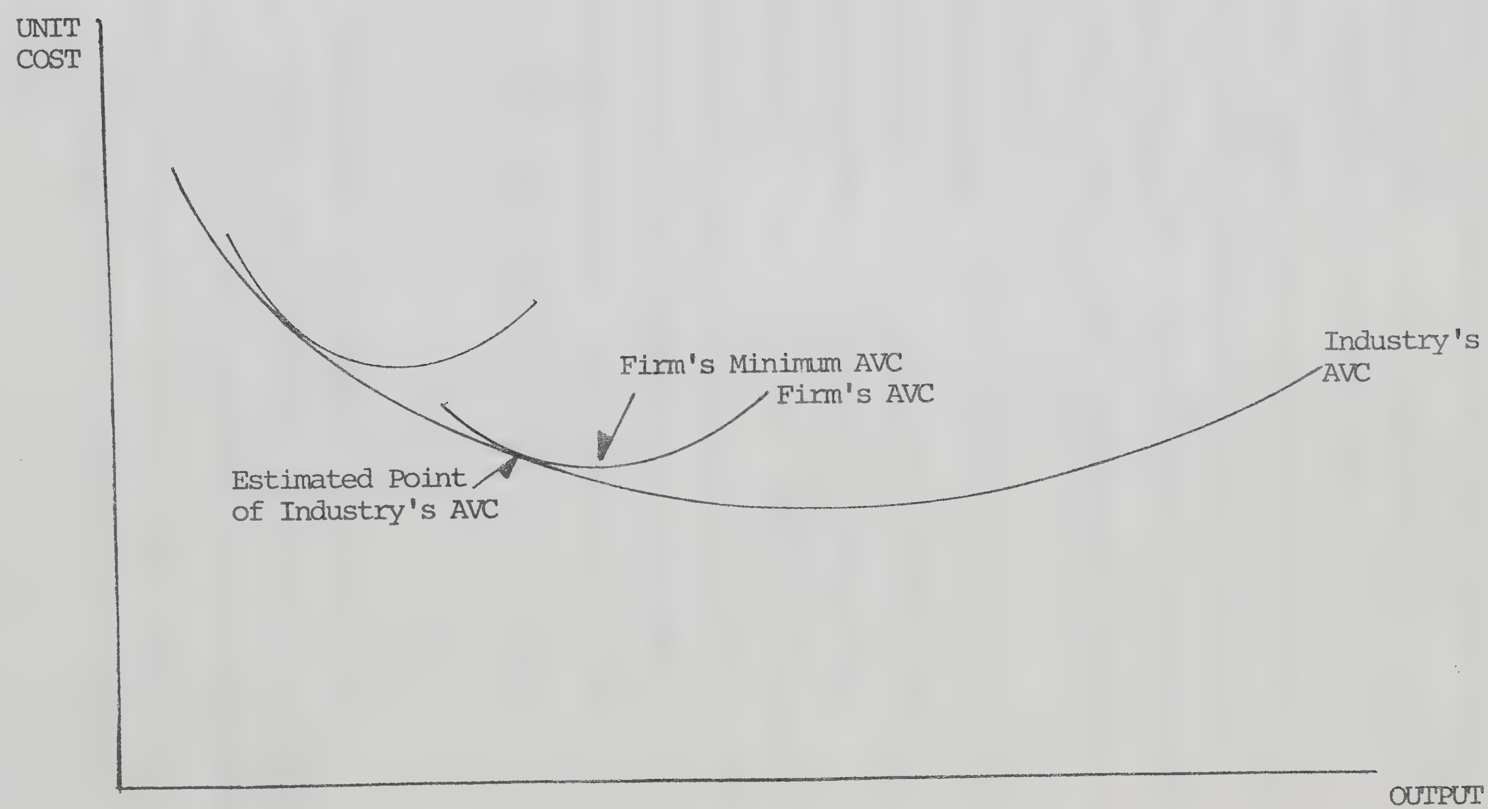
region. Since each sawmill has its own average variable cost curve (see Figure 7.2), the null hypothesis that the sawmills have reached the flat portion of their variable cost curves, or alternatively, have no significant unexploited scale economies, can be tested against the alternative hypothesis that the null hypothesis is false. The acceptance region can be defined as (Kmenta, 1971, pp.186-191)

$$\Pr.\{SE - 1.96\ (St.\ Error) \leq SE1\} = .95\ , \tag{46}$$

where, in the above definition, a 95 percent level of confidence has been selected on a t-distribution with only the left tail being considered. Consideration of the right tail would imply a different hypothesis test that incorporates diseconomies of scale but which in the present case can be ruled out since no sawmill exhibited diseconomies of scale. The estimated value of the left hand side of the above equation defines the lower range of values of the scale economies and if this value is negative, the hypothesized value (SE1) lies below the lower range of the scale economies estimated. The null hypothesis that there are no significant unexploited scale economies cannot be rejected. In other words, the sawmill has reached the flat portion of its average variable cost curve.

Forty-five sawmills have positive values and the remaining have negative values. These negative values do not

Figure 7.2 Firm and Industry Average Variable Cost Curves



mean that the sawmills have diseconomies of scale but that they are already in the flat portion of their individual average variable cost (AVC) curves. Actually, no sawmill exhibited diseconomies of scale (see Table 7.3).

From the total output produced by the 83 sawmills in 1978, over 90 percent was produced by the 45 (54 percent) firms that exhibited some degree of economies of scale. The remaining 10 percent of output was produced by the remaining 38 (46 percent) sawmills.

Sawmills were grouped into six different size groups. The number of sawmills in each group, the median output in each group, and the economies of scale for the median sawmills are reported in Table 7.3. The results indicates a positive association between larger sawmills (higher output levels) and significant unexploited scale economies. In the smallest two size groups as much as 75 percent of the sawmills have already exhausted their scale economies.

7.4 Summary

When time series data are utilized to study the economies of scale, cost reductions due to technical change that may have occured over time are difficult to distinguish from costs that have been reduced due to scale expansion. Estimation of scale economies from cross-sectional data avoids this problem.

Scale economies estimated from a cross-sectional sample of 83 sawmills reveal that a large number of the sawmills

Table 7.3: Economies of Scale for Median Sawmill by Size Group

Size Group	1	2	3	4	5	6
Range of Output	50-200	200-680	680-850	850-2000	2000-8000	8000+
No. of Mills	19	29	5	13	5	11
Median Output	150	366	750	2000	8000	17000
Median Scale Economies	.087	.094	.104	.104	.154	.105
St. Errors of Scale Economies	.06	.058	.057	.057	.064	.068
% of Mills With Significant Scale Economies	21	28	100	100	80	91

Note output is measured in ‘000’ foot board measure.

have some amount of unexploited scale economies. The results also indicate that nearly 54 percent of the 83 sawmills have not reached the bottom point of their individual average cost. The average cost of wood output ranges from about four cents to nine cents per board foot, with lower costs generally associated with larger sawmills.

8. SUMMARY AND CONCLUSIONS

8.1 Summary and Conclusions

The problems facing the Canadian forest industry including wood supply, employment stability and competitiveness, are vast. Constraints imposed by time, funds, and experience on the part of the author necessitated selection of only one small aspect of the problem for investigation. An economic analysis of the production structure of the sawmilling industry in Alberta is the focus of research herein.

An economic analysis of a production structure generally commences by selecting statistically a functional form that captures different aspects of the production technology. A functional form that characterises a fairly complex production process is required. The characteristics of production include the estimation of substitution, own and cross-price demand elasticities, technical change biases in factor usage, productive efficiency (technical and allocative efficiency), scale effects and separability of inputs.

Duality theory enables specification of a production structure by either a production function or a cost function. The translog cost function is selected to study the production structure of the sawmilling industry in Alberta. The translog cost function is specified as a function of the input prices of labor, capital, wood, and

materials along with the output of the sawmills. The parameters of the translog cost function are estimated by utilizing both the cost function and cost share equations. Multicollinearity problems are likely to occur in the estimation process. In the present study multicollinearity was detected using the auxiliary regression approach. Elimination of two variables, square of both time and output improved the results of the models considerably. All estimated cost functions satisfied the neoclassical requirements of positivity, monotonicity and convexity. Time-series (1959-1981) and cross-sectional (1978 survey results for 83 sawmills) data sets were utilized to study different aspects of the production technology. The cross-sectional and the time-series data have identical inputs except that materials, which was used in the time-series data, was not available in the cross-sectional data set and was replaced by energy as a separate input. The measurement of the variables in the two samples also differs. Labor input is measured in manhours paid in the time series sample whereas it is measured in manmonths in the cross-sectional sample. The more preferred measure is manhours worked, which in both cases was not available. In Alberta, softwood is by far the main wood input in the sawmilling industry. To obtain the price of wood, the expenditure on softwood was divided by the volume of softwood purchased. In the cross-sectional sample the price of wood at the millgate was used.

The material price index was calculated as a Divisia index consisting of nine different types of energy inputs and other inputs such as nails, staples, kraft paper, packaging materials, etc. Finally the price of capital services was calculated as the discrete Divisia index of machinery and equipment and construction and building capital. In the cross-sectional sample the price of capital is the value of maintenance plus depreciation costs divided by the replacement value of the capital stock. The price of energy in the cross-sectional sample was calculated as energy expenditure divided by the total gigajoules of energy consumed by each sampled sawmill.

In the time series, labor shows the most significant price increase relative to that of the other inputs of capital, wood and material over the time period. The prices of other inputs have also increased during the 23 year period but not as much as that of labor. The cost share of capital registered the most variation, partly reflecting the variations in the tax rate and the interest (or the bond) rate which by all means have not remained constant.

Several translog models were generated in a sequential order of restrictiveness. Due to time and cost limitations, not all models that could be generated from the nonhomothetic translog cost function were estimated. Ordered sequential testing of nested models circumvented this problem. The maintained hypothesis is that the production structure in the sawmilling industry is nonhomothetic. In

all cases the null hypothesis of nonhomotheticity could not be rejected at reasonable levels of significance against a variety of alternative hypotheses for both data sets used. The testing of the hypotheses which involved linear restrictions on the nonhomothetic model made use of likelihood ratio test statistics. Those that involved nonlinear restrictions utilized the Wald statistics.

The nonhomothetic translog multi-input cost function is therefore selected as providing a reasonable approximation to the true underlying production technology that characterizes the sawmilling industry in Alberta. The acceptance of the nonhomothetic structure implies the acceptance of several aspects of the production process. First the cost minimizing expansion path of the sawmills is nonlinear. Stated differently the marginal rate of technical substitution among input pairs is not constant along the cost minimizing expansion path. Second, economies of scale are not constant and, given the nature of the nonhomothetic model, sawmills are constrained by relative factor prices along with the levels of output in expanding their scale of operations.

Another result that may be distilled from the nonhomothetic production structure is the fact that partial elasticities of substitution are not constant and vary over time. Labor and capital are substitutes. Labor displays a complementarity relationship with wood. Between capital and material, a complementarity relationship is also noted. Wood

and material are found to be substitutes. Examination of the estimated elasticities reveals that, at least in the case of labor and capital, the substitution relationship has been declining steadily over the years, perhaps reflecting the difficulty of substituting labor for capital as labor skills have improved over time. The use of terminal elasticity estimates may be preferable for prediction purposes, especially if such elasticities are changing over time. Use of the mean values alone may not always provide the best current picture of such elasticities.

Own-price derived demand elasticity estimates reveal that the lowest negative value exists for wood input, thus indicating the 'basic good' nature of wood input in the sawmilling production process. The derived demand for labor was elastic.

Technical change bias is labor and material savings , capital using and wood neutral. The preliminary results, based on the Cobb-Douglas cost frontier, reveal that the level of technical inefficiency averages around 40 percent in the sawmilling industry when all sawmills are assumed to be allocatively efficient. The cost frontier is the locus of the minimum cost points and technical inefficiency is relative to this locus. Second, about 80 percent of the variation in observed costs is due to technical inefficiency alone and the remaining 20 percent can be attributed to factors that are beyond the control of the sawmills. The scale economy at the frontier displays increasing returns to

scale. The results on technical inefficiency are, however, very preliminiary and without further research, should not be carried too far.

Attempts to study scale economies from temporal models can lead to confusion, especially when a temporal model contains time as proxy for technical change as in the present study. The confusion arises from difficulties in separating out cost reductions due to technical change and scale effects. To avoid this confusion, scale economies were estimated using cross-sectional data. Cross-sectional data were obtained for a sample of sawmills in Alberta from a Canadian Forestry Service survey conducted in 1978. Model selection for the cross-sectional data reveals a nonhomothetic production structure in the sawmills. Economies of scale estimates for the 83 firms reveal that only 54 percent of the sawmills have scale economies that are statistically significant. Also this 54 percent of the sawmills produced over 90 percent of the total output of the 83 sawmills.

8.2 Implications for Policy

A detailed analysis was made of the production structure of the sawmilling industry in Alberta. The objective of making such an analysis arose from the broader problems of wood supply, employment generation, and competitiveness facing the Canadian forestry sector in general.

What is known about the sawmilling industry in Alberta are some of the following. A large stock of old growth forest which is accessible and economically operatable exists but is uncommitted. Over 50 percent of the sawmilling industry's output finds its way into foreign markets.

The economic analysis revealed that wood-using technical change bias and scale effects are both neutral. Wood and labor were found to be complements, indicating that more wood utilization enhances employment. However, the wood using technical change is neutral on the one hand, and on the other, capital is a substitute for both labor and wood. It appears that measures to improve the skills of the workers in this industry are likely to have beneficial effects in terms of higher productivity and increased employment. Improvement in skills is likely to exacerbate further the already declining trend observed in the labor capital substitution relationship. Furthermore, such a policy in turn may enhance wood utilization as well.

Scale economies provide some information on the reorganisation aspects of an industry. If many of the smaller sawmills had exhibited diseconomies of scale, then discouraging continuance of such sawmills could be a possible action policy makers could take if such action improved overall efficiency in the industry. More of the larger sawmills had scale economies than the smaller sawmills. The unexploited scale economies in the larger sawmills were less than one percent. Therefore, to encourage

larger sawmills and discourage smaller sawmills may not be desirable. First, such a policy is likely to increase unemployment resulting from the labor capital substitution relationship that will take place in the larger sawmills and will add to the displaced labor from the smaller sawmills. Second, increased efficiency gains resulting from mill expansion may not be large enough to compensate the income lost by the displaced workers, given that the level of unexploited scale economies in the larger sawmills are very low. However, further research is needed to determine the efficiency gain that may result if smaller sawmills are displaced and should be compared with the income lost by the displaced workers.

8.3 Suggestions for Future Research

In many sectors of the economy, differences between production and other (owners, proprietors and clerical staff) workers exist. These differences may be reflected in working hours, working conditions as well the type of on-the-job training and other factors. Aggregating these two labor types on the grounds that they are homogeneous may not be appropriate. A test for such aggregation would be of interest. Other workers may need to be specified as a separate input in the cost functions. Relationships of the types examined in this research could then be analysed with respect to other workers which may have implications for employment.

Another area of further research for policy purposes is the possible development of an aggregate cost function for the sawmilling industry across Canada. If such a cost function exists, then the effect of policy change on the sawmilling industry across the country could be anticipiated. However, if such a function does not exist, then different policies may have to be formulated for different provinces to meet the desired goals. Such a study may provide insights into stablity issues in the industry across the country.

In the province a large number of smaller sawmills produce a relatively low volume of output. These smaller sawmills did not exhibit diseconomies of scale. Further research on the smaller sawmills should perhaps be directed toward understanding more about these sawmills. A larger number of these sawmills are owner-operated mills, that may not readily leave the industry for reasons such as continued utilization of fully depreciated equipment and adaptability to small high quality timber tracts. With regard to the larger sawmills research should perhaps be directed more toward international trade and design of policies that improve the competitiveness of the larger sawmills.

Further research in the area of stochastic frontier functions can prove to be useful for planning purposes. The possibility of identifying mills that are associated with the cost frontier may be useful in predicting alternate mill sizes.

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10. APPENDIX

Table 10.1: Input Price Indexes (1971=100) for the Alberta Sawmilling Industry, 1959-1981.

		Price Index		
Year	Labour	Materials	Wood	Capital
1959	0.43796	0.54731	0.91134	0.41586
1960	0.45466	0.54732	0.75166	0.43031
1961	0.46922	0.54731	0.61275	0.46095
1962	0.48862	0.54731	0.47668	0.59837
1963	0.50857	0.54731	0.65405	0.44352
1964	0.50947	0.54732	0.54380	0.51469
1965	0.54352	0.60568	0.56001	0.59220
1966	0.60432	0.61309	0.67203	0.71968
1967	0.66679	0.84884	0.78864	0.77264
1968	0.72791	0.84884	0.85888	0.58485
1969	0.78324	0.89176	0.91555	0.69907
1970	0.88572	0.96212	0.89810	0.85579
1971	1.00000	1.00000	1.00000	1.00000
1972	1.62760	1.04205	1.15269	0.79949
1973	1.39173	1.13038	1.37392	1.01219
1974	1.51422	1.35127	1.47711	1.57762
1975	1.68811	1.60492	1.39449	1.43341
1976	1.94725	1.74891	1.62528	1.32920
1977	2.21644	1.75896	1.50935	1.30046
1978	2.48575	1.84796	1.69148	1.70925
1979	2.73731	2.02477	1.79584	1.93151
1980	3.17821	2.20593	1.85403	1.92033
1981	3.78586	2.33273	2.10453	2.29905

Table 10.2: Proportional Input Shares for the Alberta Sawmilling Industry, 1959-1981.

	Input Share			
Year	Labour	Materials	Wood	Capital
1959	0.132914	0.248673	0.586676	0.031737
1960	0.126738	0.313052	0.505867	0.054343
1961	0.132443	0.224928	0.573165	0.069462
1962	0.215855	0.225260	0.483844	0.075040
1963	0.196789	0.221442	0.523559	0.058209
1964	0.191654	0.416678	0.348060	0.043608
1965	0.202533	0.398672	0.344887	0.053909
1966	0.197291	0.353923	0.390268	0.058518
1967	0.202747	0.300664	0.448095	0.048494
1968	0.203239	0.357296	0.392946	0.046518
1969	0.181846	0.308100	0.464407	0.045648
1970	0.172275	0.325761	0.446874	0.055090
1971	0.163761	0.319751	0.455085	0.061405
1972	0.161479	0.314936	0.451436	0.072149
1973	0.147945	0.281694	0.482081	0.088281
1974	0.142020	0.268733	0.433029	0.156219
1975	0.160567	0.294074	0.320183	0.225175
1976	0.151291	0.276971	0.400166	0.171572
1977	0.168468	0.303086	0.381621	0.146825
1978	0.160665	0.277052	0.383847	0.178435
1979	0.154512	0.269098	0.387141	0.189249
1980	0.159978	0.297941	0.338677	0.203405
1981	0.146977	0.239008	0.427906	0.186109

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